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SET THEORY APPLIED IN TECHNOLOGICAL PARADIGMS AND TRAJECTORIES

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ABSTRACT
The purpose of this study is to integrate set theory as a novel mathematical framework into the field of evolutionary and neo-Schumpeterian economics, to analyse the intricate web of influences on technological paradigms. By employing the language of set theory, we can delineate the technological paradigm and its various institutional and knowledge-based *supportive forces* as subsets of the Universal Set of the Paradigm. With the universal set of a paradigm and the subsets within it, the aim is to provide a framework to represent a paradigm and its nuances, as outlining the spectrum of interactions between subsets. Importantly, while this paper provides a structured approach to paradigm analysis, the adaptability of this tool also enables the creation of other scenarios, encouraging further application to different technological analyses. The aim is to offer a simple mathematical tool to enhance the theoretical and practical understanding of technological evolution, facilitating an analysis of different prisms of paradigm and trajectories concepts.

INTRODUCTION

The main proposal of this paper is the application of set theory to the concept of technological progress. This approach could simplify the understanding of complexities inherent in the development of technological paradigms.
Technological progress refers to the process of developing new technologies and methods to enhance productivity, efficiency and economic output. It requires innovation, multiple counterparty interrelationship types, Research & Development, diffusion and adoption of new technologies and more. There are numerous concepts and authors involved with endogenous technological progress research, analysing how to best achieve technological progress and the approaches followed here are the neo-Schumpeterian and Evolutionary economics.
The attempt to map the technological evolution of a particular paradigm, may be more feasible with the aid of set theory instruments, where the paradigm and the *forces* related to the evolution of the paradigm will be represented as subsets of a universal set.

The application of the universal set concept aims to illustrate the following: the primary catalysts for a paradigm's expansion; the diverse nature of the *forces* that contribute to the paradigm's development; the various types of relationships between these forces and the paradigm; the existence of unexplored *forces* that can contribute to a paradigm expansion; R&D areas that can be shared by multiple counterparties; how trajectories guide the flow of knowledge in order to broaden the boundaries of a paradigm.
Set theory will be employed to represent and map the paradigm, tracing the forces related to its expansion and the eventual behaviour of its trajectories.

CHAPTER 1 Theoretical background in Paradigms and Trajectories: Evolutionary and Neo-Schumpeterian approaches

The evolutionary and neo-Schumpeterian approaches, differing from the neoclassical domain, are central to this article. These scholars share significant common ground, as highlighted by Nelson & Winter (1982, p.39), "we are evolutionary theorists for the sake of being neo-Schumpeterians." Such perspectives are chosen for their unified emphasis on innovation and technological change as primary drivers of economic progress, along with the pivotal role of knowledge and learning in the technology development environment.

One of the main landmarks of the foundations of evolutionary economics were Nelson & Winter (1982), with a perspective that views the economy as a dynamic and evolving system, suggesting that economic processes are better understood as being driven by gradual and cumulative changes, rather than equilibrium states. The *neo-Schumpeterian perspective*, built upon the foundational ideas of Schumpeter, provides a contemporary interpretation of innovation in economic change. This perspective expands beyond Schumpeter's original focus on the individual entrepreneur, by acknowledging the critical contributions of public institutions, such as universities and government bodies. These entities play a crucial role in pursuing long-term and high-risk research projects that private sectors may avoid, facilitating technology transfers and the implementation of regulatory policies designed to foster innovation. They also conduct or sponsor foundational research, which lays the groundwork for technological paradigm development. Public institutions are instrumental in endogenous technology development.

Several factors can influence the progression of a technological paradigm, including infrastructure, access to resources, entrepreneurship, market demand, regulation and others. Yet, as Lundvall & Johnson (1994) articulated, knowledge can be the most critical resource, forming the backbone of a paradigm, directly influencing its capacity to evolve and expand. Knowledge serves as the foundation upon which new ideas are generated, explored and developed into tangible innovations. It not only facilitates the development of new technologies within a paradigm, but also supports their transfer and diffusion. The expansion of knowledge, resulting from the learning processes of consumers, producers and institutions, directly influences technological progress, thus, economic change and growth.

Central to the creation, expansion and application of knowledge is creative intelligence. A concept that Nelson & Winter (1982) used to denote the cognitive processes, essential for decision-making and problem-solving within the technological progress environment. Creative intelligence fuels the generation and refinement of knowledge, driving innovation and enabling the continuous development and adaptation of technologies. Through these cognitive processes knowledge is not only formed but also dynamically evolves, supporting the ongoing advancement of technological paradigms. Creative intelligence is the driving force behind the creation and application of knowledge, lending a more dynamic and holistic aspect to economic science.

Giovanni Dosi (1982), with the concepts of technological paradigms and trajectories, embedded a framework to understand the patterns and directions of technical change. A technological paradigm outlines the main technological challenges and complexities of a specific technology field. The trajectory signals promising R&D directions within the paradigm, highlighting the most successful technological solutions at a given time. In essence, the paradigm represents the cutting-edge technologies or the specific knowledge limits of a problem-solving model to a certain technology. The problem-solving model provides the most effective answers to the challenges addressed by the current technology.

A technological trajectory was described by Dosi (1982) as the path that technological progress follows within a paradigm, becoming the pattern of the common problem-solving activity and the indication of future trends. Our interpretation suggests that trajectories will also seek knowledge sources out of the paradigm, emphasizing the dynamism and unpredictability of technological progress. A trajectory refers to the incremental and novel innovations that advance a paradigm, echoing the evolutionary and neo-Schumpeterian view of technological evolution characterized by continuity, adaptation and gradual development. If a paradigm represents the necessary knowledge in a scientific field, then trajectories are the streams through which this knowledge flows.

The complexity of a paradigm is magnified from the intricate interplay of multiple influential factors that drive the progress, including scientific research, technological advancements, regulatory marks, societal needs, market dynamics, environmental considerations and different interrelationships styles among counterparties. These factors can steer the direction of technological development in unpredictable ways, causing paradigms to evolve in response to shifts in these influential elements, marking progress by a high level of uncertainty.

As we navigate through the intricate landscape of technological innovation, the perceptions of paradigms and trajectories emerge not merely as abstract concepts, but as tangible guides illuminating the path of progress. The paradigm serves as a roadmap, delineating the contours of current technological capabilities and challenges, while trajectories provide the direction of the roadmap, revealing the potential directions for future advancements. At the core of this dynamic interplays the creative intelligence, the driver of new innovations and solutions. This intricate dance of knowledge, innovation and foresight, prepares the ground for the application of set theory in our ongoing exploration. Moving forward, the discussion will pivot to how set theory can facilitate the articulation of these complex relationships, offering a structured and nuanced perspective of the technological evolution roadmap and the directions it may take.

CHAPTER 2 Set Theory applied in technological Paradigms and Trajectories: An Evolutionary & Neo-Schumpeterian Approach

As we venture into the exploration of the complex and dynamic landscape of technological paradigms and trajectories, it might be opportune is counselled to be equipped with an analytical framework that is as multifaceted and adaptive as the subject itself. The set theory will provide an ideal mathematical and conceptual language to represent the diversity and dynamic that faces a technological paradigm, without dwelling into mathematics.

The development of set theory is attributed to several mathematicians, but Georg Cantor is often credited as the initiator of the field as a formal mathematical discipline in the 1870s and 1880s (Morris 2018). The set theory had a very intricate academic discussion around paradoxes of certain axioms, as the well-known Russel's paradox, which pointed out a contradiction in the way sets were formally defined. The Russell's paradoxes were developed by Bertrand Russell, David Hilbert and Friedrich Frege from "Principles of Mathematics", published in 1903 (Bernard, 2002). The lack of formalism and the reliance on intuitive understanding were referred as main inconsistencies of the naïve set theory. The paradoxes played a large role in the development of modern set theory, as required more rigorous definitions and rules to avoid contradictions (Fraenkel, et al 1973).

Universal Set of a Paradigm

In the realm of set theory, a universal set (U Set) is comprised of objects, which are elements or subsets[1]. An element does not contain additional objects within themselves, the subsets, differently, contain other elements and/or subsets. This distinction is pivotal, as it underpins the foundational structure of set theory (Fraenkel, et al., 1973).

Within the context of technological paradigms, elements are understood as static components, such as the quantity of a natural resource necessary for the advancement of a given paradigm. An illustrative example is a particular type of ceramic required for the fabrication of 5G antennas (Hill et al, 2021), a vital element for the development of the mobile network paradigm.

In the same context, the subsets are interpreted as systems, which are the main objects within the universal set of a paradigm. The subsets are recognised here as the technological paradigm and the *forces* affecting the evolution of this paradigm. This interpretation of subsets as systems arises from their complex nature, characterized by a dense network of interrelationships and their inherent intricacy. When viewed as systems, these subsets are delineated not solely by their constituent components, but also by the processes and interactions that unfold within and among them. This systematic viewpoint highlights that no subset operates in isolation within the universal set of a paradigm. Rather, each system is interconnected with others, constituting a dynamic and interdependent network. Such a network is instrumental in facilitating the exchange of knowledge and resources, thereby, propelling the evolution of technology within and across various systems. As noted previously, knowledge is considered here as the key ingredient of the paradigm evolution.

In relation to the paradoxes of the naïve set theory, one notable issue involves the universal set concept, which is postulated as a set that contains all sets, including itself (Hrbacek & Jech, 2017). Russell's paradox has demonstrated the inconsistency of a set containing itself, which is one of the main premises of Universal Set in Naïve Set Theory.

[1] The term subset is interchangeably called in the literature as sets.

This paradox was addressed by the development of more rigorous axiomatic set theories, like Zermelo-Fraenkel (Z-F) model, which avoid certain issues by precisely defining the properties and limitations of sets, allowing, per example, a set or subset to be an object of another set without leading to contradictions (Fraenkel, et al., 1973). Even ignoring the more rigorous axioms, the universal set presented here avoids Russel paradox, as it does not include the premise of the universal set as a subset of itself. Instead, it includes only subsets, which are the paradigm and the *forces* related to the evolution of the paradigm, given a certain moment in time. The number of objects within a universal set of a paradigm can be unknow but is limited and finite.

The universal set is one of the main concepts that bridges set theory to technological paradigm, as it has the capacity to organize the relation between distinct elements and subsets within its domain (Hrbacek & Jech, 2017). In the framework presented here, the Universal set will have a central subset – representative of a specific technological paradigm - and supporting sets, which constitute the *forces* contributing to the advancement of the central subset/paradigm. The central subset comprises the knowledge, routines, practices and other factors directly pertinent to the specific paradigm, whereas the supporting sets include similar components, albeit not only related to the specific paradigm. The supporting sets can be from a variety of sources, such as: natural science, other paradigms, regulation, government, institutions, environment and far others. This list is not exhaustive, as each paradigm will relate to different subsets to seek and exchange support. Also, supporting subsets, representing the *forces* that influence the paradigm, can act against it, as regulation that can either promote technological evolution or refrain it.

The model proposed is well-defined, with finite cardinality, thereby avoiding the pitfalls of Russell's Paradox, but considers that there are two types of supporting sets: known and unexplored. The latter will be further elaborated, but it echoes the evolutionary economics perspective that eschews the notion of perfect knowledge and static equilibrium. Consequently, this model recognizes that technological paradigms are dynamic, continuously evolving entities where new elements and subsets/systems emerge over time.

In essence, the universal set facilitates the gathering of all objects (subsets and elements) pertinent to a specific moment in the technological progression of a paradigm, organizing them within a logical framework. It serves to define the boundaries among the various systems surrounding a paradigm, facilitating certain analyses. However, it is crucial to note that the configuration of the universal set is always context-dependent, altering in response to the specific paradigm under consideration. This aligns with what was highlighted in Dosi (1982, p. 148) when stated that each paradigm will differ from other paradigms: “We shall argue also that each "technological paradigm" defines its own concept of "progress" based on its specific technological and economic trade-offs.”

Universal Set of a Paradigm and Venn Diagrams

Venn diagrams, a foundational tool in set theory, elucidate the interactions between sets through graphical representation. When applied with the concept of a Universal Set, these diagrams visually depict the relationships within a given framework. In Venn diagrams, subsets are typically illustrated by circles, while the Universal Set itself is represented by a rectangle (Brown et al, 2011). This notation allows a clear distinction between the subsets that constitute the universal set.

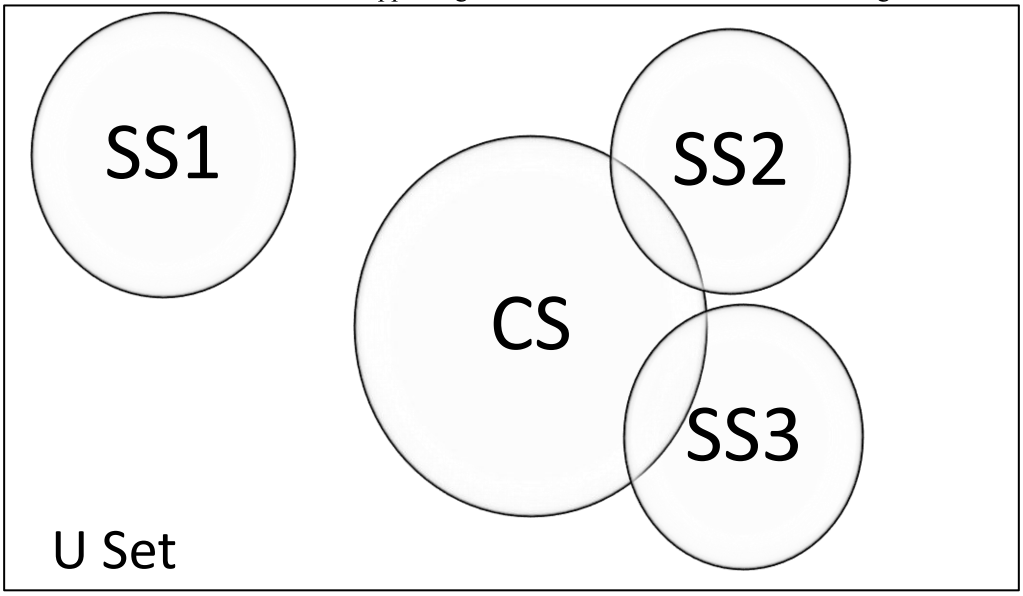
From the perspective of technological paradigms, Venn diagrams facilitate the graphical formalization of the interplay between various *forces*/subsets associated with a paradigm and the intricate relationships among them. As represented in the Figures of this section, the Universal Set aims to encapsulate all possible subsets relevant to the paradigm under discussion. In the model established here, the Universal Set contains a Central Subset (CS), which symbolizes the paradigm itself and adjacent to the CS are the Supporting Sets (SS*n*), representing the diverse forces that influence the boundaries and development of the paradigm.

This visual and notational approach, leveraging the principles of set theory and the illustrative capacity of Venn diagrams, not only underscores the holistic nature of a paradigm within its broader universe, but also facilitates a comprehensive understanding of the interconnectedness and mutual influences among its various components. By mapping out the structural and functional dynamics within these diagrams, a deeper insight into the factors driving the evolution of technological paradigms is made accessible, thereby, enabling a more nuanced and integrated perspective on their development and impact.

Within this framework, the index “*n*” in SS*n,* signifies the extensive range of potential Supporting Subsets, each of which plays a crucial role in the evolution of the paradigm. These subsets involve a wide range of influences/*forces*, including, but not limited to: natural science; other paradigms; institutional frameworks, as highlighted earlier from the *neo-Schumpeterian perspective;* investor decision-making processes; environment; regulatory bodies, whose stance can significantly sway technological progress either in favour or against technological evolution and the foundational principles of natural science that inherently guide technological advancements across all fields.

To visually summarize these relationships, the following figure will present the Universal Set of a Paradigm, featuring the Central Subset (CS) alongside with three representative Supporting Subsets (SS1, SS2, SS3), in a simplified model to aid understanding.

FIGURE 1 – Central Subset and Supporting Subsets of a Universal Set of a Paradigm



Within the realm of technological progress, the utility of Venn diagrams becomes evident as they offer an intuitive mean to delineate the intricate ecosystem of a paradigm. The figure reveals that SS1 does not intersect with CS ($SS1 \cap CS = \emptyset$), indicating that while SS1 contributes to the paradigm, it remains unaffected by its evolution. In contrast, SS2 and SS3 intersect with CS, [$(SS2 \; \& \; SS3) \cap CS \neq \emptyset$], highlighting a symbiotic relationship, thus, these Supporting Subsets not only influence but are also influenced by the Central Subset. It is important to note that the specific arrangement and interrelations between the CS and its Supporting Subsets will vary across different universal sets of technological paradigms, reflecting the unique nature of each paradigm's ecosystem.

The empirical application of Figure 1 requires a more in-depth analysis, however, for a practical illustration of these dynamics, consider the offshore oil exploration paradigm (CS) interfacing with three distinct Supporting Sets: quantum computing science (SS1), virtual reality paradigm (SS2) and seismic paradigm (SS3). Quantum Computing (QC) may not benefit directly from offshore exploration technology development, while virtual reality and seismic technology have bidirectional benefits and contributions.

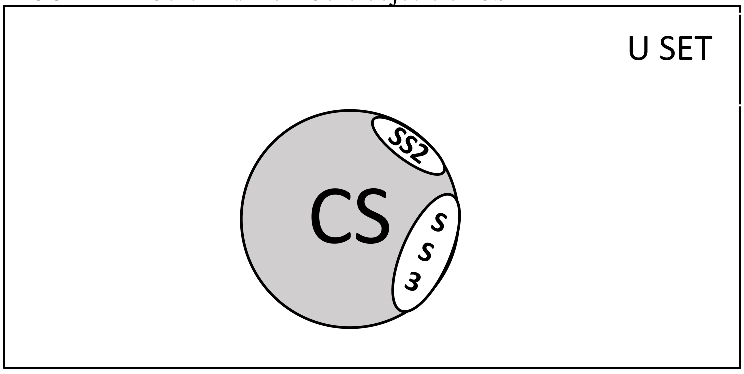
Quantum Computing (QC) holds transformative potential for numerous industries, as its ability to handle complex computations at unprecedented speeds can revolutionize how we approach problem-solving in fields characterized by vast datasets and intricate modelling requirements, as offshore oil exploration (Li et al, 2015; Elijah et al 2021). Furthermore, although not directly benefiting from advancements of the CS, SS1 can emerge as a foundational pillar, capable of developing the CS through its unparalleled computational prowess. QC holds the promise to enhance, per example, the intricate process of mapping geological formations in deep sea water regions and locating hydrocarbons areas (Fainstein et al, 2019; Abu et al, 2013), particularly those in even more challenging areas beneath pre-salt layers, where tectonic movements are more often (Hudec & Jackson, 2007)[2]. Additionally, for drilling operations, as it involves numerous variables, QC could offer optimization in solving complex mathematical models faster than traditional computing (Islam et al, 2020; Rieffel, E., & Polak). By harnessing QC, the offshore oil exploration industry could achieve significant gains in accuracy, efficiency and safety, paving the way for more sustainable and profitable operations, but QC hardly can benefit from the evolution of the offshore oil exploration paradigm.

In contrast, virtual reality (SS2) and seismic technologies (SS3) represent a bidirectional relationship with offshore oil exploration (CS), forming a symbiotic connection generated from knowledge spillovers that could allow mutual R&D endeavours. Virtual reality, used for simulating drilling environments, training personnel and visualizing geological data, can contribute to offshore exploration and also benefit from it. The demands and challenges of offshore oil exploration can provide innovations within the virtual reality paradigm, creating a cycle of innovation. This could be reflected from oil platforms adapting immersive virtual reality technology, such as the CAVE (Cave Automatic Virtual Environment), to better meet the needs of the offshore oil industry, creating knowledge spillover and developing the virtual reality paradigm (Santos at al, 2016). Similarly, seismic technology, essential for underwater mapping and guiding drilling, also benefit from advanced R&D applied in the offshore environments, as evidenced by the collaborative technology agreement between Norwergian technology company and Brazilian offshore company Petrobras. As a result of this R&D joint venture, a new seismic technology was created, advancing the realm of seismic solution (Teodoro, 2023; Skopljak, 2023). This mutual interplay exemplifies how shared challenges and advancements can foster collaboration and growth across technological fields.

[2] The mapping and location technology also depends on the seismic technology (Kumar, 2022; Alsalmi & Dossary 2023), represented by SS3, demonstrating the dynamics and multiple nuances of technological progress.

Before diving into the intricacies of relationships between subsets within the Universal Set, it might be possible to distinguish the inner components of a paradigm from the external forces. From this perspective, it is susceptible to understand that there are core and non-core objects within the CS. Core technologies are those that are fundamentally developed and utilized within the paradigm itself, without significant external influence or application. In contrast, non-core technologies require external expertise and influences to be developed and applied. The difference can be subtle and crucially depends on the extent to which a technology or knowledge area relies on external inputs. This distinction requires paradigm-specific studies to uncover the differentiation between knowledge developed internally and externally.

FIGURE 2 – Core and Non-Core objects of CS



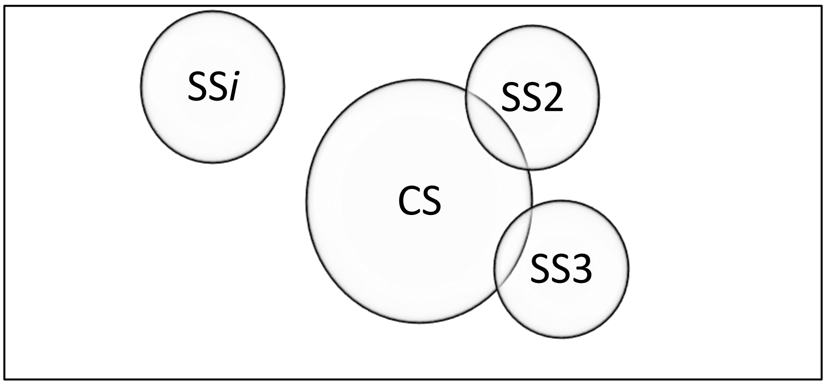
From Figure 1, it was identified that [(SS2 and SS3) ∩ CS ≠ ∅], indicating bidirectional interactions between the Central Set (CS) and the Supporting Sets (SS2 and SS3). Moving to Figure 2, which focuses on the internal structure of the CS, it is possible to discern between the core and non-core elements, represented visually by grey and white areas, respectively. A technology within the paradigm that does not intersect significantly with any external supporting set is classified as a core component of the CS.

In the context of offshore exploration, a prime example of such core technology would be the Christmas Tree technology. This technology is essential for controlling the flow of hydrocarbon (oil & gas) in subsea wells, hence, applied without any spillover outside of offshore exploration. This technology is used to control, design, manufacture and test the offshore exploration, enabling the safe and efficient management of production operations (Chen et al. 2012).

The technological specificity and lack of external applicability reinforce the delineation between core and non-core elements within the CS. In essence, while some or most of technologies within a paradigm are engaged with external *forces*, the core technologies are shaped by internal operations only. The R&D related to core technology, **from the grey area**, will not have other paradigms as supporting subset, so probably the knowledge development will behave differently from non-core technology development.

Transitioning from the internal focus of the CS to its external relationships, Figure 3 revisits and expands upon the framework introduced in Figure 1. A refined classification for Supporting Sets is introduced to better delineate their connections with CS. Specifically, Supporting Sets without interaction with CS are to be denoted as SS*i*, where "*i*" represents the index of supporting sets without an intersection with the CS, or CS ∩ SSi = ∅. Conversely, Supporting Sets intersecting with the CS are represented as SSn (SS2 and SS3), establishing a clear distinction, as CS ∩ SSn ≠ ∅. Figure 3 aims to visually articulate these relationships, distinguishing between the Supporting Subsets that bidirectionally influence the paradigm (SS*n*) and those that have a unidirectional influence (SS*i*).

FIGURE 3 – Supporting Subsets with and without interconnection [3]

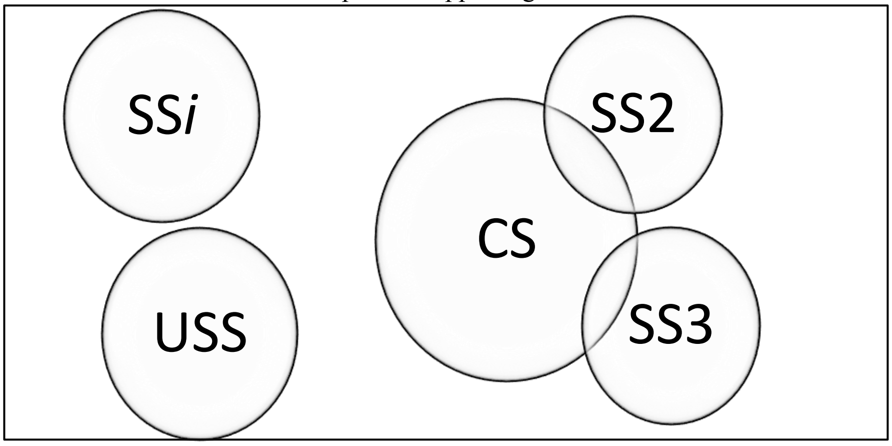


[3] The notation of "U Set" is removed for better visibility

The supporting forces that have no intersection with the paradigm (CS) will be called as SSi, on the other hand, the ones with intersection are to be treated as SSn, which are denoted here as SS2 and SS3.

Beyond the known supporting sets of CS - SSi and SSn -, lies a realm of unexplored support, which could consist of unknown or underutilized knowledge. This not yet explored support, as it is not currently integrated or recognized within the paradigm, is referenced here as Unexplored Supporting Sets (USS). This subset holds the potential to greatly influence the paradigm over time, as significant support could emerge from it as knowledge expands.

FIGURE 4 – Known and Unexplored Supporting Subsets

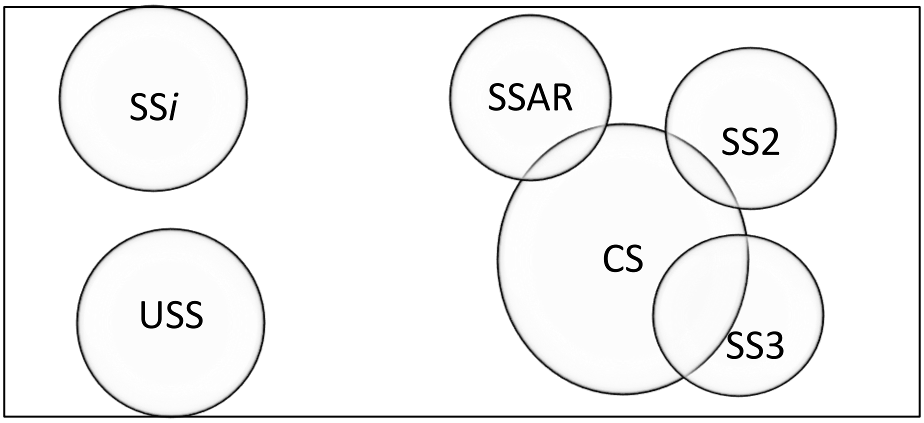


The Unexplored Supporting Subsets (USS), composed by the same objects of any other set (elements and subsets), represent a frontier of unused potential. The identification of such sources is more challengeable, but acknowledging and displaying them in a Universal Set, could help triggering the cognitive process/Creative intelligence of researchers to seek solutions not yet fully explored or not yet experimented.

A potential example of unexplored systemic subset could be Artificial Intelligence, as it stands poised to become a pivotal support for numerous technological paradigms, nevertheless, AI's contribution might currently be limited by the extent of its development. This example, though hypothetical, underscores the potential of existing technologies to become pivotal supporting systems once recognized and appropriately applied. The unveiling of the unexplored or unknown depends on time and knowledge evolution. The trajectories will discuss the unexplored subsets in further, as it is part of the trajectory role to expect the unknown and aim for solutions out of current scope or research.

Proceeding with this mapping exercise, the goal could shift towards identifying the known supporting sets within the Universal Set of a given paradigm in a certain time. It's imperative to acknowledge that the environment in which a technological paradigm flourishes is often shaped by regulatory frameworks. Thus, regulation emerges as an intrinsic component of a paradigm's landscape, influencing the development and implementation of a technology. In recognition of this, the subsequent Figure 5 will introduce the Supporting Set of Regulation (SSAR) as an essential supporting subset intertwined with the Central Set (CS), illustrating the multifaceted environment of a paradigm's universal set.

FIGURE 5 – Universal Set with SS*n,* SS*i*, CS, SSAR and USS with single overlap



The exercise of defining the supporting sets of a central set could be in favour of answering the questions raised in Dosi (1982, p.148) - "Are there regularities in the process of generation of new technologies and in technical progress thereafter? Is there any regularity in the functional relationship between the vast number of economic, social, institutional, scientific factors which are likely to influence the innovative process?". Some of the factors likely to influence the innovative process will be possible to be represented as supporting sets, hence, set theory could be used as a tool to identify regularities in the process of generation and expansion of a paradigm.

The above figure integrates the Supporting Set of Regulation (SSAR) as a critical component that continuously shapes the ecosystem of technological paradigms. The inclusion of SSAR as a part of the Supporting Subsets (SSn) underscores the permanence of regulatory influence across all paradigms, emphasizing the dynamic interplay between technological evolution and regulatory frameworks.

Each paradigm will have their own universal set, hence, further specific studies are needed to define the relationship type between a specific paradigm and regulation. The degree of interdependence between the Central Set (CS) and the Supporting Set of Regulation (SSAR) will manifest a bidirectional relationship (CS ∩ SSAR ≠ ∅), when SSAR is part of SS*n,* or alternatively and less probable, the relationship is unidirectional and there is no intersection when SSAR is part of SS*i*. However, when cross-referencing this scenario through a neo-Schumpeterian prism, the relationship might be symbiotic, thereafter, regulation should not only affect technology development, but the reciprocity should also be true (Wiener, 2004; Bezzina & Terrab 2005; Mandel, 2016).

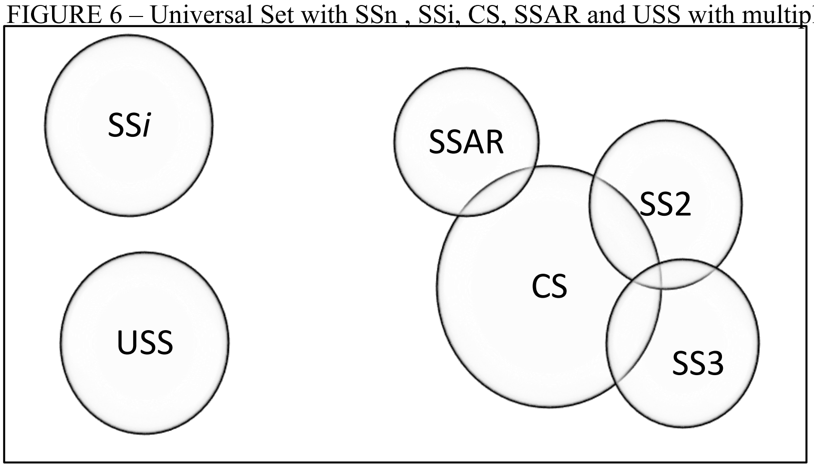
For government bodies and regulatory agencies, this model can assist in crafting adaptive regulation policies. By visualizing the complete paradigm, regulators can better understand the complex system and its various components, leading to more informed and effective regulatory decisions. It can also help identify gaps in the current regulatory framework and anticipate areas that may require further regulation. Consequently, this proactive stance enables policymakers to stay ahead of the curve, adapting regulations to foster technological innovation while ensuring safety, efficiency and public welfare.

The nomination of the SSAR subset derives from the concept of Adaptive Regulation, which by definition, regulators should adapt to technological advancements and promote development, consequently, in industries where the intersection between CS and regulation is non-empty (CS ∩ SSAR ≠ ∅), it indicates a regulatory framework aligned with the concept of adaptive regulation.

For example, the sandbox initiatives in the telecommunications industry, across various countries, exemplify the interdependent/symbiotic relationship between technology and regulation, where the regulatory bodies allow innovators to test new products, services and business models in a controlled environment without immediately having to comply with all existing regulations. This adaptive regulatory stance not only fosters innovation, but also ensures that emerging technologies can be integrated into society in a way that maximizes benefits, while mitigating risks and increasing stakeholder engagement. (Attrey et al 2020; Schaefer, 2016; Mitra 2023, Blais & Wagner, 2007)

To finalize the initial comprehensive visualization and construction of a Universal Set of a technological paradigm, Figure 6 aims to elaborate further on the possible types of interconnections within this ecosystem, showcasing a nuanced overlap between three subsets.

FIGURE 6 – Universal Set with SSn , SSi, CS, SSAR and USS with multiple overlaps



This figure, closely mirroring Figure 5, introduces a nuanced difference: SS2 and SS3, beyond their interconnection with CS, also exhibit a mutual relation, creating a **specialized area**. This area, denoted as CS ∩ SS2 ∩ SS3, holds the potential to influence the development of three distinct systems. Identifying this integrated area could become a crucial objective for trajectories aiming to harness such synergistic intersections.

Reutilizing the example in which CS is the offshore exploration, SS2 as virtual reality paradigm and SS3 as seismic technology paradigm, the intersection could potentially reflect the use of virtual reality to simulate and manage offshore oil exploration operations, enhanced by seismic for real-time data collection and environmental monitoring, potentially allowing a more efficient, safe, and effective exploration activities.

However, pinpointing such areas, necessitate a more in-depth exploration across various paradigms and probably relates to a cutting-edge technology. Additionally to this and depending of the perspective to be analysed, this specialized area could be interpreted as the concept of technological niche (Kemp et al, 1998; Scot & Geels, 2007; Stuart & Podolny 1996). As per Scot & Geels, 2007 "the challenge for future work is to build a theory as to the way in which technical change drives its own transformation by the persistent creation of new niches". In the offshore industry example, the technical change demanded in the specialized or multi-intersected area, would be driving its own transformation by creating a new niche.

Figures 5 and 6 together offer a foundational portrayal of the subsets forming the Universal Set of a technological paradigm. This portrayal is not static; it is unique to each paradigm and subject to evolution over time, reflecting the dynamic nature of technological change.

<u>Hasse diagrams & Technological trajectories</u>

In this journey, through the analysis of technological paradigms, set theory has been applied to shed light on the intricate relationships between the central paradigm (CS) and its supporting sources, also serving to distinguish known and unknown sources/subsets; core from non-core technologies; indicate potential multi collaborative research, as pointed in the intersected areas, etc.

However, the discussion has yet to cover the specific interactions between CS and the subsets without intersected areas in Venn Diagram, namely SS*i* and USS. To address this complexity, the focus shifts to technological trajectories, instead of paradigms. A technological trajectory, once defined within a paradigm, dictates the innovation's direction and pace, it embodies both, a pattern of past developments and a projection into potential future technological progress and trends. This foresight function of trajectories highlights their critical role in sculpting the future landscape of technological paradigms, illuminating areas suitable for innovation and steering research efforts towards bridging the existing knowledge gaps. Recognising the paradigm as a repository of knowledge for solving technical challenges, trajectories then represent the dynamic flow of this knowledge.

To represent the dynamics of the trajectories and to demonstrate how the subsets USS and SS*i* are connected with CS, the Hasse diagrams approach was implemented. This approach involves the application of arrows to signify these relationships, a technique , however, not conventionally utilized in traditional set theory. In Hasse diagrams the arrows indicate the relationships between subsets, the aim is to visually articulate the connections and demonstrate the potential order between various objects of a set (Dean, 2015). Although the proposition here differs from Hasse diagrams, it adopts a similar principle of visually representing relationships and additionally to this, arrows symbolise well trends and even paths that trajectories represent. The arrows can indicate that a relation exists between Set A and Set B, even in cases where their intersection is empty, providing valuable insights into the intricated dynamics that shape the evolutionary economic phenomena of a paradigm.

FIGURE 7 – Set theory and Arrows representing paradigms and trajectories

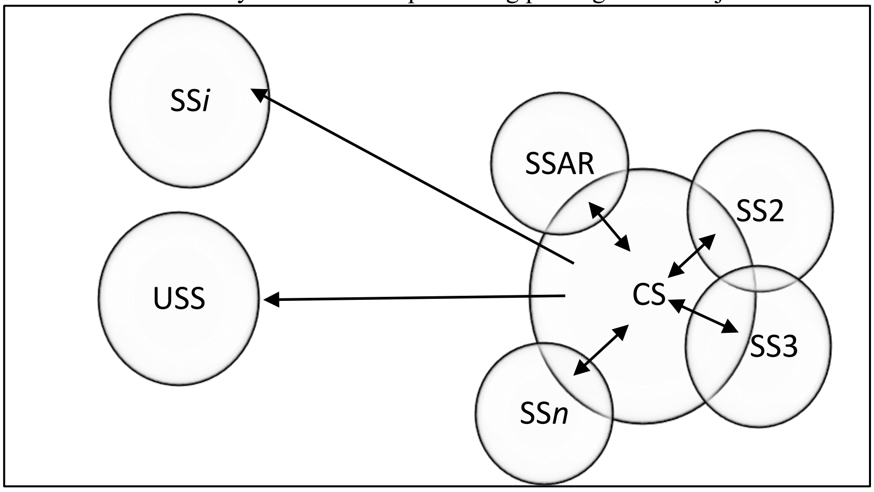


Figure 7 stands as the visual culmination of this analysis and closely mirrors the structure presented in Figure 6, however, it has *SSn* representing any other supporting subset and *introduces* the advancements of the integration of arrows relating the CS with all other subsets to enforce the relationship type between them. These arrows represent the directional flow of trajectories and interrelation types that exist among subsets. While the location of the arrows can be disregarded, their directionality is of paramount importance.

In the case of intersected supporting sets, the arrows are bidirectional, denoted by lines with arrowheads on both ends, indicating a mutual or symbiotic dependency between these subsets and CS. This symbolizes an intertwined evolution, underscoring the deep interconnectivity that fosters mutual growth and development. Conversely, for SS*i* and USS, the arrows are unidirectional, with a single arrowhead pointing away from CS, denoting a one-way dependency.

Crucially, the reach of the arrow also holds significance. Unlike other subsets, the unidirectional arrow towards USS does not fully extend to the subset, but instead suggests a potential direction of the trajectory. Unlike other subsets where arrows "enter" and establish a connection, the USS remains out of reach, representing the magnetic allure of novel and yet-to-be-explored knowledge. This absence of a direct connection does not denote irrelevance, but rather the potential for future integration as areas of unknown or subtilized technology become more known and supportive.
When the trajectory identifies and locks onto what is needed, the arrow transitions from indicating to directly connecting with the newly discovered subset/source. This shift from the unknown to the known perspective, underscoring the dynamic nature of technological evolution, where today's uncharted territories become tomorrow's foundations for innovation. As additional objects (subsets acting as systems) are identified and incorporated, the tapestry of technological development intrinsically grows more complex, reflecting the evolving nature of technological progress.

The trajectory is defined here as arrows, symbolising the dynamic principles and directions of a trajectory when seeking to find solutions that broaden the paradigm. The trajectory maps the evolution of a paradigm (CS) from its core objects to the external sources (SS*i* , SS*n, USS*). Accepting this representation also implies that the level of uncertainty, even if marginally, would decrease, as the arrows project the trends and paths a paradigm will follow.

CONCLUSIVE THOUGHTS

The general scope of this study is to better understand the cardinality and systemic structure of the universal set of a paradigm and how it can provide a clearer and more holistic picture of the paradigm's current state. With set theory tools, technological progress can be visualized, allowing for better identification of the drivers and dynamics of the technological progress surrounding a paradigm. This holistic understanding of the paradigm can guide strategic decisions and R&D choices by facilitating a more nuanced understanding of technological paradigms. It can identify the supporting sources of a paradigm and their interrelationships, including underexplored or overlapping sources. This can stimulate interdisciplinary research, fostering innovation and advancements for the paradigm.

Set theory, through the study of technological paradigms and trajectories, can be a useful tool in formalizing and analysing the dynamics of technological progress, providing an intuitive way to map how a paradigm transforms and expands. This intuitive and simple approach can be used in various situations, from professors in classrooms to agents (such as those involved in R&D and decision-making roles) when identifying trends indicated by the technological trajectory. By having a central set and various supporting sets, agents can experiment with different ideas and approaches, moving subsets and arrows on the map to explore new insights.

As this is the first application of set theory language to technological progress concepts, it is too early to draw definitive conclusions. Further research is needed to develop and refine the language used here and to verify its effectiveness in mapping paradigms and trajectories. Further studies are required not only to achieve better adherence to overall neo-Schumpeterian and evolutionary perspectives, but also to experiment set theory in different contexts of technological progress. This could involve understanding what specialized intersecting areas represent or even applying set theory to other concepts.

The advantage of set theory is its flexibility, allowing it to be adapted to different applications as required in evolutionary and neo-Schumpeterian perspectives.

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