

## EFFICIENCY SCORES ANALYSIS OF COAL MINES USING IRS, DRS AND CROSS EFFICIENCY MODELS

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### **Abstract: -**

*Economic growth world over is driven by energy, whether in the form of finite resources such as coal, oil and gas or in renewable forms such as hydroelectric, wind, solar and bio-mass or its converted form, electricity(power). Increased energy consumption (especially of electricity) is inevitable with higher GDP growth. Coal was created by the fossilised remains of plants and has high carbon content.*

*DEA is a multi-factor productivity analysis model for measuring the relative efficiency of a homogenous set of coal mines (DMU's). For every inefficient coal mine, DEA identifies a set of corresponding efficient coal mines that can be utilized as benchmarks for improvement of performance and productivity.*

*Benchmarking and ranking of coal mines based on efficiency scores using advanced DEA models like, Increasing Returns to Scale (IRS), Decreasing Returns to Scale (DRS), Cross Efficiency (CE) Models.*

**Keywords: -** Efficiency, ranking, peer group, target production



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## INTRODUCTION

The analysis carried out using TORA and DEA Software's. The analysis also carried out of OC Mines using some of the advanced

DEA Models as follows:

1. Increasing Returns Scale (IRS) Model
2. Decreasing Returns Scale (IRS) Model
3. Cross Efficiency (CE) Model

### Increasing Returns Scale (IRS) and Decreasing Returns Scale (DRS) Models

Returns to scale refers to a technical property of production that examines changes in output subsequent to a proportional change in all inputs (where all inputs increase by a constant). The output increases by that same proportional change with input then there are constant returns to scale (CRTS), sometimes referred to simply as returns to scale. If output increases by less than that proportional change, there are decreasing returns to scale (DRS). If output increases by more than that proportion, there are increasing returns to scale (IRS).

**Example:** Where all inputs increase by a factor of 2, new values for output should be: Twice the previous output given a constant return to scale (CRTS) less than twice the previous output given a decreased return to scale (DRS) more than twice previous output given an increased return to scale (IRS).

### Cross Efficiency (CE) Model

Cross efficiency in DEA allows for effective discrimination between niche performers and good overall performers. Cross efficiency [48] score of a DMU represents how well the unit is performing with respect to the optimal weights of another DMU. A DMU that achieves high cross efficiency scores is considered to be a good overall performer to improve the discrimination power of DEA, Sexton et al (1986) first introduces the concept of a cross-efficiency measure in DEA. The basic idea is to use DEA in a peerappraisal instead of a self-appraisal, which is calculated by the CRS (constant returns to scale) model. Peer evaluation is done by constituting a cross efficiency matrix of efficiency value given to each DMUs. This technique can also identify 'overall' efficient and 'false positive' DMUs, and it selects appropriate targets for poorly performing DMUs to learn as a benchmark.

## Methodology

### Cross Efficiency Models: Aggressive and Benevolent Approaches Aggressive Model

$$\begin{aligned} \min \quad & \sum_{k=1}^s \left( v_k \sum_{i \neq p} y_{ki} \right) \\ \text{s.t.} \quad & \sum_{j=1}^m \left( u_j \sum_{i \neq p} x_{ji} \right) = 1 \\ & \sum_{k=1}^s v_k y_{ki} - \sum_{j=1}^m u_j x_{ji} \leq 0, \quad \forall i \neq p \\ & \sum_{k=1}^s v_k y_{kp} - \theta_p \sum_{j=1}^m u_j x_{jp} = 0 \\ & v_k, u_j \geq 0 \quad \forall k, j \end{aligned}$$

### Benevolent Model

$$\begin{aligned} \max \quad & \sum_{k=1}^s \left( v_k \sum_{i \neq p} y_{ki} \right) \\ \text{s.t.} \quad & \sum_{j=1}^m \left( u_j \sum_{i \neq p} x_{ji} \right) = 1 \\ & \sum_{k=1}^s v_k y_{ki} - \sum_{j=1}^m u_j x_{ji} \leq 0, \quad \forall i \neq p \\ & \sum_{k=1}^s v_k y_{kp} - \theta_p \sum_{j=1}^m u_j x_{jp} = 0 \\ & v_k, u_j \geq 0 \quad \forall k, j \end{aligned}$$

## Data collection and Analysis

For the empirical application we worked with data on a survey of 15 Open Cast (OC) mines of Singareni Collieries Company Limited (SCCL). For our analysis, we have chosen **four input variables** namely,

1. Wage Cost (In Lakhs rupees per year),
2. Store Cost (In Lakhs rupees per year),

3. OBR Cost (In Lakhs rupees per year),  
 4. Other cost (In Lakhs rupees per year) and **one output variable** namely  
 5. Production (in Lakh Tonnes per year),

**Table1: Normalized Data for Open-Cast mines**

Normalized data of OC mines					
Mines(DMU)	Wage Cost	Store Cost	OBR Cost	Other Cost	Production
OCM1	1.4159	1.3481	1.6260	1.5881	1.4980
OCM2	0.4178	0.2750	1.1271	0.6606	1.0283
OCM3	0.8347	0.3747	0.2395	0.2439	0.4547
OCM4	0.2877	0.0429	0.0886	1.4318	0.9398
OCM5	2.2116	2.7843	1.0544	1.9245	1.6182
OCM6	0.1794	0.3421	0.5946	0.3132	0.6900
OCM7	0.0900	0.0640	0.1193	0.0033	0.1348
OCM8	0.8788	0.6435	2.3050	0.6806	1.2584
OCM9	0.4472	0.3099	1.5266	0.3449	0.7523
OCM10	0.3140	0.1812	0.5095	0.1531	0.4167
OCM11	0.2761	0.0975	0.4884	0.2727	0.4347
OCM12	0.8668	0.4730	1.9179	0.5059	1.3427
OCM13	2.5188	3.8545	1.5713	2.2644	2.1494
OCM14	1.7423	1.7183	0.7791	0.7015	0.8720
OCM15	2.5188	2.4909	1.0527	3.9112	1.4102

**1. Increasing Returns Scale (IRS) Model**

Returns to scale refers to a technical property of production that examines changes in output subsequent to a proportional change in all inputs (where all inputs increase by a constant). If output increases by more than that proportion, there are increasing returns to scale (IRS) which is under DEA IRS and DRS Models. The analysis carried out using Input-oriented CCR data and DEA software and results are shown in table 2.

**Table2: Efficiency, Shadow values, Peer group and Peer count values after solving Input-oriented IRS Model**

DMU	Efficiency	shadow Values	Peer Group	Peer Count
OCM1	55.10%	0.577, 0.086, 6.651	OCM4,OCM6,OCM7	0
OCM2	100%	1	OCM2	5
OCM3	100%	1	OCM3	5
OCM4	100%	1	OCM4	6
OCM5	67.80%	1.234, 0.694, 3.001	OCM3,OCM4,OCM7	0
OCM6	100%	1	OCM6	4
OCM7	100%	1	OCM7	10
OCM8	71.40%	0.656, 0.129, 3.669	OCM2,OCM6,OCM7	0
OCM9	85.70%	0.431, 0.012, 2.235	OCM2,OCM6,OCM7	0
OCM10	83.40%	0.042, 1.653, 0.346	OCM2,OCM7,OCM11	0
OCM11	100%	1	OCM11	3
OCM12	96.40%	0.153, 4.39, 1.365	OCM2,OCM7,OCM11	0
OCM13	68.20%	1.094, 0.878, 6.134	OCM3,OCM4,OCM7	0
OCM14	64.30%	1.093, 0.124, 1.913	OCM3,OCM4,OCM7	0
OCM15	39.70%	0.723, 0.958, 1.34	OCM3,OCM4,OCM7	0

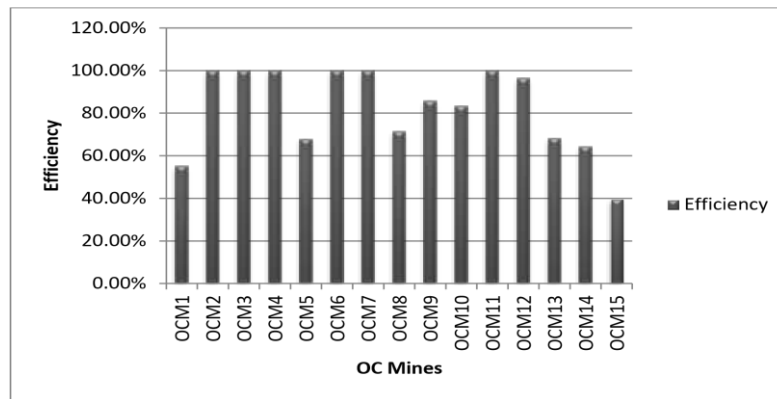
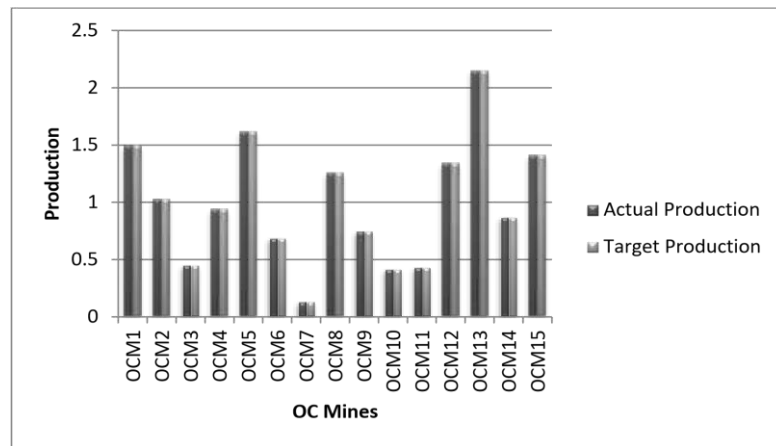


Fig 1: OC Mines Vs Efficiency score for Input-oriented IRS Model

Table 3: Improvements in Inputs and Output of OC Mines after solving Input – oriented IRS model

	Wage Cost	Store Cost	OBR Cost	Other Cost	Production
DMU	Actual to Target	Actual to Target	Actual to Target	Actual to Target	Actual to Target
OCM1	1.416 to 0.78	1.348 to 0.48	1.626 to 0.896	1.588 to 0.875	1.498 to 1.498
OCM2	0.418 to 0.418	0.275 to 0.275	1.127 to 1.127	0.661 to 0.661	1.028 to 1.028
OCM3	0.835 to 0.835	0.375 to 0.375	0.24 to 0.24	0.244 to 0.244	0.455 to 0.455
OCM4	0.288 to 0.288	0.043 to 0.043	0.089 to 0.089	1.432 to 1.432	0.94 to 0.94
OCM5	2.212 to 1.5	2.784 to 0.684	1.054 to 0.715	1.924 to 1.305	1.618 to 1.618
OCM6	0.179 to 0.179	0.342 to 0.342	0.595 to 0.595	0.313 to 0.313	0.69 to 0.69
OCM7	0.09 to 0.09	0.064 to 0.064	0.119 to 0.119	0.003 to 0.003	0.135 to 0.135
OCM8	0.879 to 0.627	0.644 to 0.459	2.305 to 1.254	0.681 to 0.486	1.258 to 1.258
OCM9	0.447 to 0.383	0.31 to 0.266	1.527 to 0.759	0.345 to 0.296	0.752 to 0.752
OCM10	0.314 to 0.262	0.181 to 0.151	0.509 to 0.414	0.153 to 0.128	0.417 to 0.417
OCM11	0.276 to 0.276	0.098 to 0.098	0.488 to 0.488	0.273 to 0.273	0.435 to 0.435
OCM12	0.867 to 0.836	0.473 to 0.456	1.918 to 1.363	0.506 to 0.488	1.343 to 1.343
OCM13	2.519 to 1.718	3.854 to 0.84	1.571 to 1.072	2.264 to 1.544	2.149 to 2.149
OCM14	1.742 to 1.121	1.718 to 0.537	0.779 to 0.501	0.702 to 0.451	0.872 to 0.872
OCM15	2.519 to 1	2.491 to 0.398	1.053 to 0.418	3.911 to 1.553	1.41 to 1.41



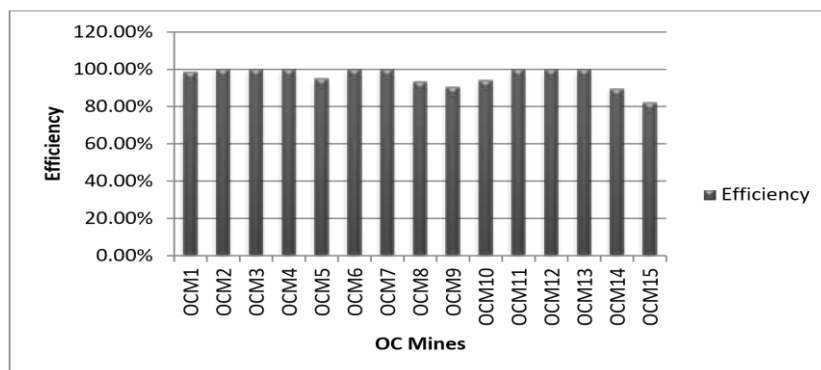
**Fig 2: Actual Production Vs Target Production for Input-oriented IRS Model**

## 2. Decreasing Returns Scale (DRS) Model

The analysis carried out using DEA software and results are shown in table 4.

**Table 4: Efficiency, Peer group and Peer count values after solving Output-oriented DRS Model**

DMU	Efficiency	Peer Group	Peer Count
OCM1	98.60%	OCM4,OCM12,OCM13	0
OCM2	100%	OCM2	3
OCM3	100%	OCM3	1
OCM4	100%	OCM4	5
OCM5	95%	OCM4,OCM6,OCM13	0
OCM6	100%	OCM6	5
OCM7	100%	OCM7	3
OCM8	93.30%	OCM12,OCM13	0
OCM9	90.50%	OCM2,OCM6,OCM7,OCM12	0
OCM10	94%	OCM2,OCM6,OCM7,OCM12	0
OCM11	100%	OCM11	1
OCM12	100%	OCM12	6
OCM13	100%	OCM13	6
OCM14	89.40%	OCM4,OCM6,OCM13	0
OCM15	82.10%	OCM4,OCM12,OCM13	0

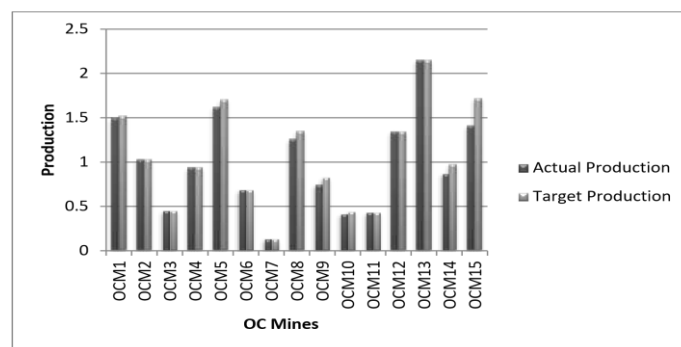


**Fig 3: OC Mines Vs Efficiency score for Output-oriented DRS Model**

The target production of in-efficient mines is increased drastically by adjusting slack variable in concerned input value and results are shown in table 5.

**Table 5: Improvements in Inputs and Output of OC Mines after solving Output – oriented DRS model**

	Wage Cost	Store Cost	OBR Cost	Other Cost	Production
DMU	Actual to Target	Actual to Target	Actual to Target	Actual to Target	Actual to Target
OCM1	1.416 to 1.254	1.348 to 1.348	1.626 to 1.626	1.588 to 1.085	1.498 to 1.519
OCM2	0.418 to 0.418	0.275 to 0.275	1.127 to 1.127	0.661 to 0.661	1.028 to 1.028
OCM3	0.835 to 0.835	0.375 to 0.375	0.24 to 0.24	0.244 to 0.244	0.455 to 0.455
OCM4	0.288 to 0.288	0.043 to 0.043	0.089 to 0.089	1.432 to 1.432	0.94 to 0.94
OCM5	2.212 to 1.71	2.784 to 2.49	1.054 to 1.054	1.924 to 1.924	1.618 to 1.704
OCM6	0.179 to 0.179	0.342 to 0.342	0.595 to 0.595	0.313 to 0.313	0.69 to 0.69
OCM7	0.09 to 0.09	0.064 to 0.064	0.119 to 0.119	0.003 to 0.003	0.135 to 0.135
OCM8	0.879 to 0.879	0.644 to 0.498	2.305 to 1.915	0.681 to 0.519	1.258 to 1.349
OCM9	0.447 to 0.447	0.31 to 0.31	1.527 to 1.038	0.345 to 0.345	0.752 to 0.831
OCM10	0.314 to 0.236	0.181 to 0.181	0.509 to 0.509	0.153 to 0.153	0.417 to 0.443
OCM11	0.276 to 0.276	0.098 to 0.098	0.488 to 0.488	0.273 to 0.273	0.435 to 0.435
OCM12	0.867 to 0.867	0.473 to 0.473	1.918 to 1.918	0.506 to 0.506	1.343 to 1.343
OCM13	2.519 to 2.519	3.854 to 3.854	1.571 to 1.571	2.264 to 2.264	2.149 to 2.149
OCM14	1.742 to 0.634	1.718 to 1.02	0.779 to 0.779	0.702 to 0.702	0.872 to 0.975
OCM15	2.519 to 1.723	2.491 to 2.491	1.053 to 1.053	3.911 to 1.959	1.41 to 1.719

**Fig 4: Actual Production Vs Target Production for Output-oriented DRS Model**

### 3. Cross Efficiency (CE) Model

This problem solved using algorithm 7 and DEAP software, the results are shown in table 6.

**Table 6: Results produced after solving Cross Efficiency model for 10 OC Mines**

	Efficiency	OCM 1	OCM 2	OCM 3	OCM 4	OCM 5	OCM 6	OCM 7	OCM 8	OCM 9	OCM10
OCM1	55.09	55.09	74.48	63.5	100	56.55	100	100	50.47	49.16	68.11
OCM2	100	47.17	100	45.58	100	31.89	100	100	67.75	76.07	80.55
OCM3	100	54.67	64.07	100	100	67.82	83.8	100	43.43	40.7	64.04
OCM4	100	55.09	74.48	63.5	100	56.55	100	100	50.47	49.16	68.11
OCM5	67.82	54.67	64.07	100	100	67.82	83.8	100	43.43	40.7	64.04
OCM6	100	55.09	74.48	63.5	100	56.55	100	100	50.47	49.16	68.11
OCM7	100	47.17	100	45.58	100	31.89	100	100	67.75	76.07	80.55
OCM8	71.4	43.03	100	38.96	78.19	28.07	100	100	71.4	85.69	80.05
OCM9	85.69	43.03	100	38.96	78.19	28.07	100	100	71.4	85.69	80.05
OCM10	83.42	40.44	100	43.41	82.58	24.95	84.43	100	70.48	85.52	83.42

### Conclusions

From table 2 OCM7 referred 10 times as a peer count is most efficient unit in all aspects and used as a referring mine for other mines to improve their productivity based on this IRS analysis. OCM15 shown very poor performance is 39.70%. If output increases by less than that proportional change, there are decreasing returns to scale (DRS) which is described in 3.6 under DEA IRS and DRS Models. The analysis carried out using DEA software and results are shown in table 4. Lot of improvement in efficiency scores shown in DRS model solved using Output- oriented CCR model. OCM12 and OCM13 got maximum peer count of 6 that means these two mines referred maximum number of times for other in-efficient mines for improving their performance. Cross efficiency in DEA allows for effective discrimination between niche performers and good overall performers. Cross efficiency score of a DMU represents how well the unit is performing with respect to the optimal weights of another DMU.

Here we have solved only 10 OC Mines due to space problem existed in width wise of efficiencies for representing in rows wise and column wise for effective comparison with efficiencies of other in-efficient mines

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