GENERALIZED SOFT INTERSECTIONAL IDEALS IN TERNARY SEMIRINGS

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Abstract: -

In this paper we introduce the notions of soft intersectional ternary subsemirings and soft intersectional ideals in ternary semirings. We also discuss some basic results associated with these notions. In the last part of the paper, we characterize regular and weakly regular ternary semirings by their soft intersectional ideals.

Key words: Ternary semirings, regular ternary semirings, weakly regular ternary semirings, soft intersectional ideals in ternary semirings.

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1. INTRODUCTION

Molodtsov,[24] in 1999, came forward with a novel mathematical tool (named, soft sets) to analyze the uncertain situations, where complete information are not available and accentuated the need of usage of soft sets in various fields with interesting examples. In the second phase, after Molodtsov, the names of P. K. Maji, M. I. Ali, M.

Shabir, F. Feng, Y. B. Jun, H. Aktas, N. Cagman, A. Sezgin and A. O. Atagun are of vital importance in the development of soft logic to make it able enough to be used in applied fields, see [3, 5, 6, 8, 9, 12, 13, 14, 15, 16, 22, 23, 25, 26, 29].

The idea of semirings have been focal point of study by mathematicians and has set its worth in information sciences. The role of ideals in semirings is of cardinal importance and serve greatly in numerous objects. H. S. Vandiver[28] primarily generated the structure of semirings. J. Ahsan's[2] introduction and characterization of weakly regular semirings was coined in terms of their ideals.

D. H. Lehmer[17] was first introduced ternary algebraic structure in 1932, called triplexes. Next, in 1971, Lister[18] characterized the additive semigroups of rings, which are closed under the triple ring product as ternary rings. Later, Duta and Kar, and Bhambri[10, 11, 7] theorized the notion of ternary semirings and weakly regular ternary semirings, respectively.

F. Feng and Y. B. Jun[12] were the pioneers to define the soft semirings and soft ideals in soft semirings, in 2008. Later, in 2014, X. Ma and J. Zhan[19, 20] contributed by their work on Hemirings through soft intersection h-ideals and h-hemiregular hemirings via (M,N)-SI-h-ideals. A good amount of work already been done on soft semigroups and soft rings[1, 4] in 2010, by U. Acar and M. I. Ali. Next, S. Z. Song and H. S. Kim[27] worked on soft intersection semigroups. After that, T.

Mahmood and U. Tariq [21] accelerated the work in the same sense by introducing soft intersection k-semirings and detailed work in this regard.

In this paper, we are going to introduce the concept of soft intersection ternary semirings. Next, we shall characterize the regular ternary semirings by means of soft intersection ternary ideals, and weakly regular ternary semirings in the same terminology.

2. Preliminaries

2.1 Definition

A set $S \neq \Phi$ with a binary operation addition "+" and a ternary multiplication ".", denoted by juxtaposition, is said to be a ternary semiring S , if it satisfies the following conditions:

$$(i) (mno)pq = m(nop)q = mn(opq)$$

$$(ii) (m+n)pq = mpq + npq$$

$$(iii) m(n+p)q = mnq + mpq$$

$$(iv) mn(p+q) = mnp + mnq,$$

for all $r, s, t, u, v \in S$. It shall be denoted by $\mathbf{T}_S$.

We can see that any semiring can be reduced to $\mathbf{T}_S$.

However, the converse may not hold in general.

2.2 Example

The set of -ve integers is $\mathbf{T}_S$ under usual "+" and ternary ".". Note that it is not semi-ring under usual "+" and ".".

2.3 Definition

An additive subsemigroup J of S is called a ternary left (right, lateral) ideal of S if $uvj (juv, u jv) \in J$, $\forall u, v \in S$ and $j \in J$. It shall be denoted by $\mathbf{T}_{Li}$ ($\mathbf{T}_{Ri}$, $\mathbf{T}_{Ei}$).

If J is both $\mathbf{T}_{Li}$ and $\mathbf{T}_{Ri}$ of S , then J is called two-sided ideal of S . It shall be denoted by $\mathbf{T}_{Ti}$.

If J is an $\mathbf{T}_{Ti}$ and $\mathbf{T}_{Ei}$ of S , then J is called an ideal of S . It shall be denoted by $\mathbf{T}_I$.

2.4 Definition

$\mathbf{T}_S$ with 1 means $11u = 1u1 = u11 = u$, $\forall u \in S$.

2.5 Definition

Let $(R, +, \cdot)$ be a $\mathbf{T}_S$. An element $l \in R$ is called multiplicatively idempotent if $l^3 = l$.

2.6 Definition

S is called regular if for any $r \in S$, there exist

$u \in S$ such that $r = rur$ or $r = rSr$, $\forall r \in S$. It shall be denoted by $\mathbf{T}_{vN\text{-}r}$.

2.7 Theorem

S is regular if and only if $A \cap B \cap C = AB$ for A ,

B and C as $\mathbf{T}_{Ri}$, $\mathbf{T}_{Li}$ and $\mathbf{T}_{Ei}$ of S .

2.8 Definition

An element $a \in S$ is called right weakly (left weakly, lateral weakly) regular if $a \in (aS)^3 S$ ($a \in S(aS)^3$, $a \in SSaSaSS$), respectively.

A ternary semiring S is called right weakly (left weakly, lateral weakly) regular if all its elements are right weakly (left weakly, lateral weakly) regular, respectively. It shall be denoted by $\mathbf{T}_{RW}$ r - ($\mathbf{T}_{LW}$ r - , $\mathbf{T}_{EW}$ r -).

From now to onward, if otherwise stated, S will always denote a $\mathbf{T}_S$. Further, for undefined terms and notions of S , see [17].

If U is initial universe, E is a set of parameters and $A, B, C, \dots$ are subsets of E . Then we have:

2.9 Definition [24]

A soft set (φ, A) over U means that φ is a mapping $\varphi: A \rightarrow P(U)$. Then we will write here (φ, A, U) instead of writing " (φ, A) is soft set of A over U ", if otherwise stated.

2.10 Definition [27]

Let (φ, A, U) and $\rho \in P(U)$. Then the set $i_A(\varphi, \rho) = \{w \in A : \rho \subseteq \varphi(w)\}$ is called ρ -inclusive set of (φ, A) .

2.11 Definition

For $\Phi \neq A \subseteq S$, the characteristic soft set of S

over U is $C_A(w) = \begin{cases} \Phi & \text{if } w \notin A, \\ U & \text{if } w \in A. \end{cases}$ C_S is known as identity soft set of S over U .

3. Main Results

Here we take $E=S$, if otherwise stated.

3.1 Definition

For (φ_1, S, U) and (φ_2, S, U) , the sum $(\varphi_1 + \varphi_2, S, U)$ is defined by

$$(\varphi_1 + \varphi_2)(w) = \bigcup_{w=a+b} \{\varphi_1(a) \cap \varphi_2(b)\}, \forall w \in S.$$

3.2 Definition

For (φ_1, S, U) , (φ_2, S, U) and (φ_3, S, U) , the product

$(\varphi_1 \varphi_2 \varphi_3, S, U)$ is defined by

$$(\varphi_1 \varphi_2 \varphi_3)(w) = \bigcup_{w=a_i b_j c_l} [\bigcap_i \{\varphi_1(a_i) \cap \varphi_2(b_j) \cap \varphi_3(c_l)\}],$$

$\forall w \in S$.

In this chapter, if otherwise stated, we will use the consideration $\Phi \subseteq M \subset N \subseteq U$.

3.3 Definition

(φ, S, U) is called (M, N) soft intersectional subsemiring of S if

(i) $\varphi(r+s) \cup M \supseteq \varphi(r) \cap \varphi(s) \cap N$,

(ii) $\varphi(rst) \cup M \supseteq \varphi(r) \cap \varphi(s) \cap \varphi(t) \cap N$,

$\forall r, s, t \in S$. It will be denoted by $\varphi^t_{[M, N]_{SI-S}}$.

3.4 Definition

(φ, S, U) is called (M, N) soft intersectional left (right, lateral) ideal of S if

$$(i) \varphi(r+s) \cup M \supseteq \varphi(r) \cap \varphi(s) \cap N, ,$$

$$(ii) \varphi(rst) \cup M \supseteq \varphi(t) \cap N,$$

$$(\varphi(rst) \cup M) \supseteq \varphi(r) \cap N$$

$$\varphi(rst) \cup M \supseteq \varphi(s) \cap N \forall r, s, t \in S$$

It will be denoted by $\varphi^t_{[M,N]_{SI-L_i}}$ ($\varphi^t_{[M,N]_{SI-R_i}}$, $\varphi^t_{[M,N]_{SI-E_i}}$).

(φ, S, U) is called (M, N) soft intersectional

two-sided ideal of S , if it is $\varphi^t_{[M,N]_{SI-L_i}}$ and $\varphi^t_{[M,N]_{SI-R_i}}$

of S and denoted by $\varphi^t_{[M,N]_{SI-T_i}}$.

(φ, S, U) is called (M, N) soft intersectional ideal

of S , if it is $\varphi^t_{[M,N]_{SI-T_i}}$ and $\varphi^t_{[M,N]_{SI-E_i}}$ of S and

denoted by $\varphi^t_{[M,N]_{SI-I}}$.

If $0 \in S$. Then clearly, $\varphi(0) \cup M \supseteq \varphi(s) \cap N$ and

$$(\varphi(0) \cap N) \cup M \supseteq (\varphi(s) \cap N) \cup M, \forall s \in S.$$

3.5 Definition

For two soft intersectional sets (φ_1, S, U) and

(φ_2, S, U) ,

$$\varphi_1 \subseteq_{[X,Y]} \varphi_2 \Leftrightarrow (\varphi_1(s) \cap N) \cup M \supseteq (\varphi_2(s) \cap N) \cup M,$$

$$\forall s \in S.$$

Obviously, $\varphi_1 =_{[M,N]} \varphi_2 \Leftrightarrow \varphi_1 \subseteq_{[M,N]} \varphi_2$ and

$$\varphi_2 \subseteq_{[M,N]} \varphi_1.$$

3.6 Theorem

(φ, S, U) is $\varphi^t_{[M,N]_{SI-S}}$ of S if and only if

$$\varphi + \varphi \subseteq_{[M,N]} \varphi \text{ and } \varphi^3 \subseteq_{[M,N]} \varphi.$$

Proof: Let us assume (φ, S, U) is an $\varphi^t_{[M,N]_{SI-S}}$ of

S . Then $\forall s \in S$

$$((\varphi + \varphi)(s) \cap N) \cup M$$

$$= \bigcup_{s=m+n} [\varphi(m) \cap \varphi(n) \cap N] \cup M$$

$$= \bigcup_{s=m+n} [(\varphi(m) \cap \varphi(n) \cap N) \cap N] \cup M$$

$$\subseteq \bigcup_{s=m+n} [(\varphi(m+n) \cup M) \cap N] \cup M$$

$$= \bigcup_{s=m+n} [\varphi(s) \cap N] \cup M$$

$$= (\varphi(s) \cap N) \cup M.$$

Thus, $\varphi + \varphi \subseteq_{[M,N]} \varphi$.

$\varphi^3 \subseteq_{[M,N]} \varphi$ is analogous.

Conversely, let us assume $\varphi + \varphi \subseteq_{[M,N]} \varphi$ and

$\varphi^3 \subseteq_{[M,N]} \varphi$. Then $\forall r, s \in S$

$$\begin{aligned} (\varphi(r+s) \cap N) \cup M &\supseteq ((\varphi + \varphi)(r+s) \cap N) \cup M \\ &= \bigcup_{r+s=m+n} [\varphi(m) \cap \varphi(n) \cap N] \cup M \\ &\supseteq (\varphi(r) \cap \varphi(s) \cap N) \cup M. \end{aligned}$$

In the same pattern, one can prove

$$(\varphi(rst) \cap N) \cup M \supseteq (\varphi(r) \cap \varphi(s) \cap \varphi(t) \cap N) \cup M$$

Thus, (φ, S, U) is an φ^t of S .

$_{[M,N]SI-S}$

3.7 Lemma

(φ, S, U) is an φ^t (φ^t , φ^t) of S if

$_{[M,N]SI-L_i}$ $_{[M,N]SI-R_i}$ $_{[M,N]SI-E_i}$

and only if $\varphi + \varphi \subseteq_{[M,N]} \varphi$ and $C_S C_S \varphi \subseteq_{[M,N]} \varphi$

$(\varphi C_S C_S \subseteq_{[M,N]} \varphi, C_S \varphi C_S \subseteq_{[M,N]} \varphi)$.

Proof: Let us assume (φ, S, U) be an φ^t of S .

$_{[M,N]SI-L_i}$

Then, by Theorem 3.6, we have $\varphi + \varphi \subseteq_{[M,N]} \varphi$

$\forall r \in S$.

Next, $((C_S C_S \varphi)(r) \cap N) \cup M$

$$\begin{aligned} &= \bigcup_{r=\sum_{i=1}^n m_i n_i o_i} \{ \bigcap_i [C_S(m_i) \cap C_S(n_i) \cap \varphi(o_i)] \cap N \} \cup M \\ &\subseteq \bigcup_{r=\sum_{i=1}^n m_i n_i o_i} \{ \bigcap_i [(U \cap N) \cap (\varphi(\sum_{i=1}^n m_i n_i o_i) \cap N)] \\ &\quad \cap N \} \cup M \\ &\subseteq \bigcup_{r=\sum_{i=1}^n m_i n_i o_i} (\varphi(r) \cap N) \cup M \\ &= (\varphi(r) \cap N) \cup M. \end{aligned}$$

Thus, $C_S C_S \varphi \subseteq_{[M,N]} \varphi$.

Conversely, let us assume $\varphi + \varphi \subseteq_{[M,N]} \varphi$ and

$C_S C_S \varphi \subseteq_{[M,N]} \varphi$. Then, by Theorem 3.6, we have

$$(\varphi(r+t) \cap N) \cup M \supseteq (\varphi(r) \cap \varphi(t) \cap N) \cup M$$

$\forall r, t \in S$.

Next,

$$\begin{aligned} &(\varphi(rst) \cap N) \cup M \\ &\supseteq ((C_S C_S \varphi)(rst) \cap N) \cup M \\ &= \bigcup_{rst=\sum_{i=1}^n m_i n_i o_i} \{ \bigcap_i [C_S(m_i) \cap C_S(n_i) \cap \varphi(o_i)] \\ &\quad \cap N \} \cup M \\ &\supseteq [C_S(r) \cap C_S(s) \cap \varphi(t) \cap N] \cup M \\ &= [U \cap U \cap \varphi(t) \cap N] \cup M \\ &= (\varphi(t) \cap N) \cup M \end{aligned}$$

Hence, ψ be a φ^t of S .

$_{[M,N]SI-L_i}$

3.8 Theorem

$\Phi \neq W \subseteq S$ is a ternary semiring of S if and only if $C_S(r)$ is an φ^t of S .
 $[M, N]_{SI-S}$

Proof: Suppose that W is a ternary semiring of S and $r, s, t \in S$.

Case-I:

If $r, s, t \in W$, then $r+t, rst \in W$. Then

$$\begin{aligned} & (C_W(r+t) \cap N) \cup M \\ &= (U \cap N) \cup M \\ &= (U \cap U \cap N) \cup M \\ &= (C_W(r) \cap C_W(t) \cap N) \cup M \text{ and} \\ & (C_W(rst) \cap N) \cup M \\ &= (U \cap N) \cup M \\ &= (U \cap U \cap U \cap N) \cup M \\ &= (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M. \end{aligned}$$

Case-II:

If at least one, say $t \notin W$, then $C_W(t) = \Phi$.

$$\begin{aligned} \text{Then } & (C_W(r+t) \cap N) \cup M \\ & \supseteq (\Phi \cap N) \cup M \\ &= (C_W(r) \cap \Phi \cap N) \cup M \\ &= C_W(r) \cap C_W(t) \cap N, \end{aligned}$$

$$\begin{aligned} \text{and } & (C_W(rst) \cap N) \cup M \\ & \supseteq (\Phi \cap N) \cup M \\ &= (C_W(r) \cap \Phi \cap C_W(t) \cap N) \cup M \\ &= (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M. \end{aligned}$$

By combining both cases

$$\begin{aligned} & (C_W(r+t) \cap N) \cup M \supseteq (C_W(r) \cap C_W(t) \cap N) \cup M \\ \text{and} \\ & (C_W(rst) \cap N) \cup M \\ & \supseteq (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M. \end{aligned}$$

Hence, C_W is an φ^t of S .
 $[M, N]_{SI-S}$

Conversely, assume that C_W is an φ^t of S
 $[M, N]_{SI-S}$

and $r, s, t \in W$. Then

$$\begin{aligned} & (C_W(r+t) \cap N) \cup M \\ & \supseteq (C_W(r) \cap C_W(t) \cap N) \cup M \\ &= (U \cap U \cap N) \cup M = (U \cap N) \cup M. \\ & \Rightarrow C_W(r+t) = U, \\ \text{and} \\ & (C_W(rst) \cap N) \cup M \\ & \supseteq (C_W(r) \cap C_W(s) \cap C_W(t) \cap N) \cup M \\ &= (U \cap U \cap U \cap N) \cup M = (U \cap N) \cup M. \\ & \Rightarrow C_W(rst) = U. \end{aligned}$$

Thus, $r+t, rst \in W$, $\forall r, s, t \in W$.

This shows that W is a φ^t of S .
 $[M, N]_{SI-S}$

3.9 Theorem

Let $\Phi \neq W \subseteq S$. Then W is a left (right, letal) ideal of S if and only if C_W is an $\varphi^t_{[M,N]_{SI-L_i}}$ of S .

Proof: Let W be a ternary left ideal of S and $r, s, t \in S$.

Case-I: For $r, t \in W$ and $t \in W$, we have

$r + t \in W$ and $rst \in W$, respectively.

Then $(C_W(r+t) \cap N) \cup M$

$$= (U \cap N) \cup M = (U \cap U \cap N) \cup M$$

$$= (C_W(r) \cap C_W(t) \cap N) \cup M,$$

and $(C_W(rst) \cap N) \cup M$

$$= (U \cap N) \cup M$$

$$= (C_W(t) \cap N) \cup M.$$

Case-II: If $t \notin W$, then $r + t \notin W$ and $rst \notin W$.

Then $(C_W(r+t) \cap N) \cup M$

$$\supseteq (\Phi \cap N) \cup M$$

$$= (C_W(r) \cap \Phi \cap N) \cup M$$

$$= (C_W(r) \cap C_W(t) \cap N) \cup M,$$

and $(C_W(rst) \cap N) \cup M$

$$\supseteq (\Phi \cap N) \cup M$$

$$= (C_W(t) \cap N) \cup M.$$

By combining the both cases, we have

$$(C_W(r+t) \cap N) \cup M \supseteq (C_W(r) \cap C_W(t) \cap N) \cup M$$

and

$$(C_W(rst) \cap N) \cup M \supseteq (C_W(t) \cap N) \cup M.$$

Hence, C_W is $\varphi^t_{[M,N]_{SI-L_i}}$ of S .

Conversely, assume that C_W is an $\varphi^t_{[M,N]_{SI-L_i}}$ of S

and $r, s, t \in W$.

Then $(C_W(v+w) \cap Y) \cup X$

$$\supseteq (C_W(v) \cap C_W(w) \cap Y) \cup X$$

$$= (U \cap U \cap Y) \cup X \subseteq (U \cap Y) \cup,$$

and $(C_W(rst) \cap N) \cup M$

$$\supseteq (C_W(t) \cap N) \cup M$$

$$= (U \cap N) \cup M.$$

Thus, $r + t \in W$ and $rst \in W$, $\forall r, t \in W$ and

$\forall t \in W$, respectively.

This shows that, W is ternary left ideal of S .

3.10 Theorem

Let (φ_1, S, U) , (φ_2, S, U) be the φ_i^t $($
 $[M, N]_{SI-L_i}, s$

φ_i^t , φ_i^t $)$ of S (where $i=1, 2$), then
 $[M, N]_{SI-R_i}, s$ $[M, N]_{SI-E_i}, s$

their sum $(\varphi_1 + \varphi_2, S, U)$ is also an

$(\varphi_1 + \varphi_2)^t_{[M, N]_{SI-L_i}}$ $((\varphi_1 + \varphi_2)^t_{[M, N]_{SI-R_i}}, (\varphi_1 + \varphi_2)^t_{[M, N]_{SI-E_i}})$
 of S .

Proof: To show that $(\varphi_1 + \varphi_2, S, U)$ is an $(\varphi_1 + \varphi_2)^t_{[M, N]_{SI-L_i}}$ of S , we will have to prove that
 $[M, N]$

(i) $((\varphi_1 + \varphi_2)(r+t) \cap N) \cup M$

$$\supseteq ((\varphi_1 + \varphi_2)(r) \cap (\varphi_1 + \varphi_2)(t) \cap N) \cup M$$

$\forall r, t \in S$.

(ii) $((\varphi_1 + \varphi_2)(rst) \cap N) \cup M$

$$\supseteq ((\varphi_1 + \varphi_2)(t) \cap N) \cup M \quad \forall r, s, t \in S.$$

Now,

$$\begin{aligned} & ((\varphi_1 + \varphi_2)(r) \cap (\varphi_1 + \varphi_2)(t) \cap N) \cup M \\ &= (\{ \bigcup_{r=m+n} [\varphi_1(m) \cap \varphi_2(n)] \} \cap \{ \bigcup_{t=p+q} [\varphi_1(p) \cap \varphi_2(q)] \}) \\ & \quad \cap N) \cup M \end{aligned}$$

$$= \bigcup_{r=m+n, t=p+q} [\varphi_1(m) \cap \varphi_2(n) \cap \varphi_1(p) \cap \varphi_2(q) \cap N] \cup M$$

$$= \bigcup_{r=m+n, t=p+q} [(\varphi_1(m) \cap \varphi_1(p) \cap N) \cap (\varphi_2(n) \cap \varphi_2(q) \cap N) \cap N] \cup M$$

$$\subseteq \bigcup_{r+t=m+p+n+q} [\varphi_1(m+p) \cap \varphi_2(n+q) \cap N] \cup M$$

$$\subseteq \bigcup_{r+t=g'+h'} [\varphi_1(g') \cap \varphi_2(h') \cap N] \cup M$$

$$= ((\varphi_1 + \varphi_2)(r+t) \cap N) \cup M,$$

and $((\varphi_1 + \varphi_2)(t) \cap N) \cup M$

$$= \bigcup_{t=u+v} [\varphi_1(u) \cap \varphi_2(v) \cap N] \cup M$$

$$= \bigcup_{t=u+v} [(\varphi_1(u) \cap \varphi_2(v) \cap N) \cap N] \cup M$$

$$\subseteq \bigcup_{rst=rsu+rsv} [\varphi_1(rsu) \cap \varphi_2(rsv) \cap N] \cup M$$

$$\subseteq \bigcup_{rst=u'+v'} [\varphi_1(u') \cap \varphi_2(v') \cap N] \cup M$$

$$= ((\varphi_1 + \varphi_2)(rst) \cap N) \cup M.$$

Hence, $(\varphi_1 + \varphi_2, S, U)$ is an $(\varphi_1 + \varphi_2)^t_{[M, N]_{SI-L_i}}$ of S .

3.11 Theorem

Let (φ_1, S, U) , (φ_2, S, U) and (φ_3, S, U) be the

φ_i^t (φ_i^t , φ_i^t) of S (where $[M, N]_{SI-L_i, S}$ $[M, N]_{SI-R_i, S}$ $[M, N]_{SI-E_i, S}$ $i=1, 2, 3$), then $(\varphi_1\varphi_2\varphi_3, S, U)$ is also an

$(\varphi_1\varphi_2\varphi_3)^t$ ($(\varphi_1\varphi_2\varphi_3)^t$, $(\varphi_1\varphi_2\varphi_3)^t$) of S .
 $[M, N]_{SI-L_i}$ $[M, N]_{SI-R_i}$ $[M, N]_{SI-E_i}$

Proof: Let $r, t \in S$. Then

$$\begin{aligned} (\varphi_1\varphi_2\varphi_3)(r) &= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{\bigcap_i [\varphi_1(m_i) \cap \varphi_2(n_i) \cap \varphi_3(o_i)]\} \\ (\varphi_1\varphi_2\varphi_3)(t) &= \bigcup_{t=\sum_{j=1}^l m'_j n'_j o'_j} \{\bigcap_j [\varphi_1(m'_j) \cap \varphi_2(n'_j) \cap \varphi_3(o'_j)]\} \end{aligned}$$

Now,

$$\begin{aligned} & ((\varphi_1\varphi_2\varphi_3)(r) \cap (\varphi_1\varphi_2\varphi_3)(t) \cap N) \cup M \\ &= \left(\bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{\bigcap_i [\varphi_1(m_i) \cap \varphi_2(n_i) \cap \varphi_3(o_i)]\} \right) \cap \\ & \left(\bigcup_{t=\sum_{j=1}^l m'_j n'_j o'_j} \{\bigcap_j [\varphi_1(m'_j) \cap \varphi_2(n'_j) \cap \varphi_3(o'_j)]\} \right) \cap \\ & N) \cup M \\ &= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i, t=\sum_{j=1}^l m'_j n'_j o'_j} [\bigcap_i \bigcap_j [\varphi_1(m_i) \cap \varphi_2(n_i) \cap \varphi_3(o_i) \cap \varphi_1(m'_j) \cap \varphi_2(n'_j) \cap \varphi_3(o'_j)] \cap N] \cup M \\ &= \bigcup_{r+t=\sum_{i=1}^k m_i n_i o_i + \sum_{j=1}^l m'_j n'_j o'_j} [\bigcap_i \bigcap_j (\varphi_1(m_i) \cap \varphi_1(m'_j) \cap N) \cap (\varphi_2(n_i) \cap \varphi_2(n'_j) \cap N) \cap (\varphi_3(o_i) \cap \varphi_3(o'_j) \cap N)] \cap N] \cup M \\ &\subseteq \bigcup_{r+t=\sum_{p=1}^q x_p y_p z_p} \{\bigcap_p [\varphi_1(x_p) \cap \varphi_2(y_p) \cap \varphi_3(z_p)] \cap N\} \cup M \\ &= ((\varphi_1\varphi_2\varphi_3)(r+t) \cap N) \cup M. \end{aligned}$$

Next, for $r, s, t \in S$

$$\begin{aligned} & ((\varphi_1\varphi_2\varphi_3)(t) \cap N) \cup M \\ &= \bigcup_{t=\sum_{i=1}^k x_i y_i z_i} \{\bigcap_i [\varphi_1(x_i) \cap \varphi_2(y_i) \cap \varphi_3(z_i)] \cap Y\} \cup X \\ &= \bigcup_{rst=\sum_{i=1}^k r s x_i y_i z_i} \{\bigcap_i [\varphi_1(r s x_i) \cap \varphi_2(y_i) \cap \varphi_3(z_i)] \cap N\} \cup M \end{aligned}$$

$$\subseteq \bigcup_{rst=\sum_{i=1}^k x'_i y'_i z'_i} \{\bigcap_i [\varphi_1(x'_i) \cap \varphi_2(y'_i) \cap \varphi_3(z'_i)] \cap N\} \cup M$$

$$= ((\varphi_1 \varphi_2 \varphi_3)(rst) \cap N) \cup M.$$

Thus, $((\varphi_1 \varphi_2 \varphi_3)(r+t) \cap N) \cup M$

$$\supseteq ((\varphi_1 \varphi_2 \varphi_3)(r) \cap (\varphi_1 \varphi_2 \varphi_3)(t) \cap N) \cup M,$$

$$\forall r, t \in S \text{ and } ((\varphi_1 \varphi_2 \varphi_3)(rst) \cap N) \cup M$$

$$\supseteq ((\varphi_1 \varphi_2 \varphi_3)(t) \cap N) \cup M, \forall r, s, t \in S.$$

Therefore, $(\varphi_1 \varphi_2 \varphi_3, S, U)$ is an $(\varphi_1 \varphi_2 \varphi_3)_{SI-L_i}^t$ of S .

3.12 Theorem

For a ternary semiring S , the following are equivalent

- (i) S is $\mathbf{T}_{vNr}$.
- (ii) $E \cap F \cap G = EFG$, for any E, F , and G as $\mathbf{T}_{Ri}$, $\mathbf{T}_{Ei}$ and $\mathbf{T}_{Li}$ of S , respectively.
- (iii) $\varphi_1 \cap \varphi_2 \cap \varphi_3 = \varphi_1 \varphi_2 \varphi_3$, for any φ_1, φ_2 and φ_3 as φ_1^t, φ_2^t and φ_3^t of S , respectively.

Proof: (i) $\Rightarrow$ (iii) Let φ_1, φ_2 and φ_3 are

$$\varphi_1^t, \varphi_2^t \text{ and } \varphi_3^t \text{ of } S,$$

respectively. Then for any $w \in S$, we have

$$((\varphi_1 \varphi_2 \varphi_3)(t) \cap N) \cup M$$

$$= \bigcup_{t=\sum_{i=1}^k m_i n_i o_i} \{\bigcap_i [\varphi_1(m_i) \cap \varphi_2(n_i) \cap \varphi_3(o_i)] \cap N\} \cup M$$

$$\subseteq \bigcup_{t=\sum_{i=1}^k m_i n_i o_i} \{\bigcap_i [\varphi_1(m_i n_i o_i) \cap \varphi_2(m_i n_i o_i) \cap \varphi_3(m_i n_i o_i)] \cap N\} \cup M$$

$$\subseteq \bigcup_{t=\sum_{i=1}^k m_i n_i o_i} \{\bigcap_i [(\varphi_1(m_i n_i o_i) \cap N) \cap (\varphi_2(m_i n_i o_i) \cap N) \cap (\varphi_3(m_i n_i o_i) \cap N)] \cap N\} \cup M$$

$$= \bigcup_{t=\sum_{i=1}^k m_i n_i o_i} [\{\bigcap_i \varphi_1(m_i n_i o_i)\} \cap \{\bigcap_i \varphi_2(m_i n_i o_i)\} \cap \{\bigcap_i \varphi_3(m_i n_i o_i)\} \cap N] \cup M$$

$$\subseteq \bigcup_{t=\sum_{i=1}^k m_i n_i o_i} [\{\varphi_1(\sum_{i=1}^k m_i n_i o_i) \cap \varphi_2(\sum_{i=1}^k m_i n_i o_i) \cap \varphi_3(\sum_{i=1}^k m_i n_i o_i)\} \cap N] \cup M$$

$$= \bigcup_{t=\sum_{i=1}^k m_i n_i o_i} \{\varphi_1(t) \cap \varphi_2(t) \cap \varphi_3(t) \cap N\} \cup M$$

$$= (\varphi_1(t) \cap \varphi_2(t) \cap \varphi_3(t) \cap N) \cup M$$

$$= ((\varphi_1 \cap \varphi_2 \cap \varphi_3)(t) \cap N) \cup M.$$

Thus,

$$(\varphi_1 \varphi_2 \varphi_3 \cap N) \cup M \subseteq (\varphi_1 \cap \varphi_2 \cap \varphi_3 \cap N) \cup M$$

Since, S is $\mathbf{F}_{\mathcal{V}N\text{-}r}$. Then for any $s \in S$, there exist

$$p \in S \text{ such that } s = sps.$$

Now,

$$((\varphi_1 \cap \varphi_2 \cap \varphi_3)(s) \cap N) \cup M$$

$$= (\varphi_1(s) \cap \varphi_2(s) \cap \varphi_3(s) \cap N) \cup M$$

$$= (\varphi_1(s) \cap (\varphi_2(s) \cap Y) \cap \varphi_3(s) \cap N) \cup M$$

$$\subseteq (\varphi_1(s) \cap \varphi_2(sps) \cap \varphi_3(s) \cap N) \cup M.$$

Thus,

$$((\varphi_1 \cap \varphi_2 \cap \varphi_3)(s) \cap N) \cup M$$

$$\subseteq \bigcup_{w=\sum_{i=1}^n x_i y_i z_i} \{ \bigcap_i [\varphi_1(x_i) \cap \varphi_2(y_i) \cap \varphi_3(z_i)] \cap N \} \cup M$$

$$= ((\varphi_1 \varphi_2 \varphi_3)(s) \cap N) \cup M.$$

This implies

$$(\varphi_1 \cap \varphi_2 \cap \varphi_3 \cap N) \cup M \subseteq (\varphi_1 \varphi_2 \varphi_3 \cap N) \cup M$$

Hence, finally we get

$$\varphi_1 \cap \varphi_2 \cap \varphi_3 =_{[M,N]} \varphi_1 \varphi_2 \varphi_3.$$

(iii) $\Rightarrow$ (ii) Assume (iii) holds. Now, let E, F and G are $\mathbf{F}_{R_i}, \mathbf{F}_{E_i}$ and $\mathbf{F}_{L_i}$ of S , respectively.

Then, the characteristic functions C_E, C_F and

$$C_G \text{ are } \varphi^t_{[M,N]_{SI-R_i}}, \varphi^t_{[M,N]_{SI-E_i}} \text{ and } \varphi^t_{[M,N]_{SI-L_i}} \text{ of } S,$$

respectively. Next, by hypothesis

$$C_{EFG} =_{[M,N]} C_E \cdot C_F \cdot C_G =_{[M,N]} C_E \cap C_F \cap C_G$$

$$\Rightarrow C_{EFG} =_{[M,N]} C_{E \cap F \cap G}$$

$$\Rightarrow EFG = E \cap F \cap G.$$

(i) $\Leftrightarrow$ (ii) is followed by Theorem 2.7.

3.13 Theorem

For a ternary semiring S with 1, the following are equivalent:

(i) S is $\mathbf{F}_{R_{\text{se}}}$.

(ii) All $\mathbf{F}_{R_i}$ of S are idempotent.

(iii) $\varphi_1 \cap \varphi_2 \cap \varphi_3 = \varphi_1 \varphi_2 \varphi_3$, for any φ_1, φ_2 and

$$\varphi_3 \text{ as } \varphi_1^t_{[M,N]_{SI-R_i}}, \varphi_2^t_{[M,N]_{SI-E_i}} \text{ and } \varphi_3^t_{[M,N]_{SI-L_i}} \text{ of } S,$$

respectively.

(iv) All $\varphi^t_{[M,N]_{SI-R_i}}$ of S are fully idempotent.

(v) $\varphi_1 \cap \varphi_2 \cap \varphi_3 = \varphi_1 \varphi_2 \varphi_3$, for any φ_1, φ_2 and φ_3 as φ_1^t , φ_2^t and φ_3^t of S ,
 $_{[M,N]_{SI-R_i}}$, $_{[M,N]_{SI-E_i}}$ and $_{[M,N]_{SI-L_i}}$
 respectively.

If S is commutative then (i)-(v) are equivalent to

(vi) S is $\mathbf{F}_{\mathcal{N} \cdot r}$.

Proof: (i) $\Rightarrow$ (iv) Let φ be an φ^t of S and
 $_{[M,N]_{SI-R_i}}$

$r \in S$. Then

$$\begin{aligned} & (\varphi^3(r) \cap N) \cup M \\ &= ((\varphi\varphi\varphi)(r) \cap N) \cup M \\ &= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{ \bigcap_i [\varphi(m_i) \cap \varphi(n_i) \cap \varphi(o_i)] \\ & \cap N \} \cup M \\ &= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{ \bigcap_i [(\varphi(m_i) \cap N) \cap \varphi(n_i) \cap \\ & \varphi(o_i)] \cap N \} \cup M \\ &\subseteq \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} \{ \bigcap_i [\varphi(m_i n_i o_i) \cap \varphi(n_i) \cap \\ & \varphi(o_i)] \cap N \} \cup M \\ &= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\{ \bigcap_i \varphi_1(m_i n_i o_i) \} \cap \{ \bigcap_i (\varphi_2(n_i) \\ & \cap \varphi_3(o_i)) \} \cap N] \cup M \\ &= \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\{ \varphi_1(\sum_{i=1}^k m_i n_i o_i) \} \cap \{ \bigcap_i (\varphi_2(n_i) \\ & \cap \varphi_3(o_i)) \} \cap N] \cup M \\ &\subseteq \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\varphi(r) \cap \{ \bigcap_i (\varphi_2(n_i) \cap \varphi_3(o_i)) \} \\ & \cap N] \cup M \\ &\subseteq \bigcup_{r=\sum_{i=1}^k m_i n_i o_i} [\varphi(r) \cap N] \cup M \\ &= (\varphi(r) \cap N) \cup M. \end{aligned}$$

This implies $\varphi^3 \subseteq_{[M,N]} \varphi$.

Since, S is $\mathbf{F}_{\mathcal{N} \cdot r}$. So, $r \in rSrSrSS$ and we can

write $r = \sum_{i=1}^k r m_i r n_i r p_i q_i$, where m_i, n_i, p_i, q_i
 $\in S$ and $k \in \mathbb{N}$.

Then

$$\begin{aligned} & (\varphi(r) \cap \varphi(r) \cap \varphi(r) \cap N) \cup M \\ &\subseteq (\varphi(r m_i r) \cap \varphi(r n_i r) \cap \varphi(r p_i q_i) \cap N) \cup M, \forall i. \\ &\Rightarrow (\varphi(r) \cap N) \cup M \\ &\subseteq \bigcap_i \{ \varphi(r m_i r) \cap \varphi(r n_i r) \cap \varphi(r p_i q_i) \cap N \} \cup M \end{aligned}$$

$$\begin{aligned}
&\subseteq \bigcup_{r=\sum_{i=1}^k rm_i r(m_i r) p_i q_i} \{\bigcap_i [\varphi(rm_i r) \cap \varphi(rm_i r) \\
&\cap \varphi(rp_i q_i)] \cap N\} \cup M \\
&\subseteq \bigcup_{r=\sum_{j=1}^l a'_j b'_j c'_j} \{\bigcap_j [\varphi(a'_j) \cap \varphi(b'_j) \cap \varphi(c'_j)] \\
&\cap N\} \cup M \\
&= ((\varphi\varphi\varphi)(r) \cap N) \cup M \\
&= (\varphi^3 \cap N) \cup M.
\end{aligned}$$

This implies $\varphi \subseteq_{[M,N]} \varphi^3$.

Thus, $\varphi =_{[M,N]} \varphi^3$. Therefore, φ is fully idempotent.

(iv) $\Rightarrow$ (i) Let $r \in S$ and $V = rSS$ be a $\mathfrak{F}_{\mathcal{R}_i}$ of S , formed by r . Then $r \in V$ and the characteristic function C_V of V is an φ^t of S . Next, by $_{[M,N]SI-\mathcal{R}_i}$

$$\text{hypothesis } C_V =_{[M,N]} C_V \cdot C_V \cdot C_V =_{[M,N]} C_{V^3}$$

$$\Rightarrow V = V^3$$

$$\Rightarrow r \in V^3 = (rSS)^3$$

$$\Rightarrow r \in rSrSrSS.$$

Thus, S is $\mathfrak{F}_{\mathcal{R}_{w,r}}$.

(i) $\Rightarrow$ (v) Let φ_1 , φ_2 and φ_3 as φ_1^t , $_{[M,N]SI-\mathcal{R}_i}$

φ_2^t and φ_3^t of S , respectively. Then for $_{[M,N]SI-E_i}$ $_{[M,N]SI-L_i}$

any $r \in S$, we have

$$\begin{aligned}
&((\varphi_1 \varphi_2 \varphi_3)(r) \cap N) \cup M \\
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} \{\bigcap_i [\varphi_1(x_i) \cap \varphi_2(y_i) \cap \varphi_3(z_i)] \\
&\cap N\} \cup M \\
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} \{\bigcap_i [\varphi_1(x_i) \cap \varphi_2(y_i) \cap \varphi_3(z_i)] \\
&\cap N\} \cup M \\
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} \{\bigcap_i [(\varphi_1(x_i) \cap N) \cap (\varphi_2(y_i) \cap N) \\
&\cap (\varphi_3(z_i) \cap N)] \cap N\} \cup M \\
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} \{\bigcap_i [\varphi_1(x_i y_i z_i) \cap \varphi_2(x_i y_i z_i) \\
&\cap \varphi_3(x_i y_i z_i)] \cap N\} \cup M \\
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} [\{\bigcap_i (\varphi_1(x_i y_i z_i))\} \cap \{\bigcap_i (\varphi_2(x_i y_i z_i))\} \\
&\cap \{\bigcap_i (\varphi_3(x_i y_i z_i))\} \cap N] \cup M
\end{aligned}$$

$$\begin{aligned}
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} [\{\varphi_1(\sum_{i=1}^k x_i y_i z_i)\} \cap \{\varphi_2(\sum_{i=1}^k x_i y_i z_i)\} \\
&\cap \{\varphi_3(\sum_{i=1}^k x_i y_i z_i)\} \cap N] \cup M \\
&= \bigcup_{r=\sum_{i=1}^k x_i y_i z_i} [\varphi_1(r) \cap \varphi_2(r) \cap \varphi_3(r) \cap N] \cup M \\
&= (\varphi_1(r) \cap \varphi_2(r) \cap \varphi_3(r) \cap N) \cup M \\
&= ((\varphi_1 \cap \varphi_2 \cap \varphi_3)(r) \cap N) \cup M.
\end{aligned}$$

This implies $\varphi_1 \varphi_2 \varphi_3 \subseteq_{[M,N]} \varphi_1 \cap \varphi_2 \cap \varphi_3$.

Since, S is $\mathbf{F}_{R_{w,s}}$. Then $r \in S$ can be written as

$$r = \sum_{i=1}^k r m_i r n_i r p_i q_i \text{ where } m_i, n_i, p_i, q_i \in S \text{ and } k \in N.$$

Now,

$$\begin{aligned}
&((\varphi_1 \cap \varphi_2 \cap \varphi_3)(r) \cap N) \cup M \\
&= (\varphi_1(r) \cap \varphi_2(r) \cap \varphi_3(r) \cap N) \cup M \\
&\subseteq (\varphi_1(r m_i r) \cap \varphi_2(r m_i r) \cap \varphi_3(r p_i q_i) \cap N) \cup M
\end{aligned}$$

Thus, we have

$$\begin{aligned}
&\subseteq ((\varphi_1 \cap \varphi_2 \cap \varphi_3)(r) \cap N) \cup M \\
&\subseteq \bigcap_i \{\varphi_1(r m_i r) \cap \varphi_2(r m_i r) \cap \varphi_3(r p_i q_i) \\
&\cap N\} \cup M
\end{aligned}$$

$$\subseteq \bigcup_{r=\sum_{i=1}^k r m_i r (r m_i r) r p_i q_i} \{\bigcap_i [\varphi_1(r m_i r) \cap \varphi_2(r m_i r)$$

$$\cap \varphi_3(r p_i q_i)] \cap N\} \cup M$$

$$\subseteq \bigcup_{r=\sum_{j=1}^k a'_j b'_j c'_j} \{\bigcap_j [\varphi_1(a'_j) \cap \varphi_2(b'_j) \cap \varphi_3(c'_j)]$$

$$\cap Y\} \cup X$$

$$= ((\varphi_1 \varphi_2 \varphi_3)(r) \cap N) \cup M.$$

$$\Rightarrow \varphi_1 \varphi_2 \varphi_3 \subseteq_{[M,N]} \varphi_1 \cap \varphi_2 \cap \varphi_3.$$

Therefore, $\varphi_1 \cap \varphi_2 \cap \varphi_3 =_{[M,N]} \varphi_1 \varphi_2 \varphi_3$.

(v) $\Rightarrow$ (iii) Let E, F and G be the $\mathbf{F}_{R_i}, \mathbf{F}_{E_i}$ and

$\mathbf{F}_{L_i}$ of S , respectively. Then, the characteristic

functions C_E, C_F and C_G are $\varphi^i_{[M,N]_{SI-R_i}}, \varphi^i_{[M,N]_{SI-E_i}}$

and $\varphi^i_{[M,N]_{SI-L_i}}$ of S , respectively. Next, by hypothesis

$$C_{EFG} =_{[M,N]} C_E \cdot C_F \cdot C_G =_{[M,N]} C_E \cap C_F \cap C_G$$

$$\Rightarrow C_{EFG} =_{[M,N]} C_E \cap C_F \cap C_G$$

$$\Rightarrow EFG =_{[M,N]} E \cap F \cap G.$$

(i) $\Rightarrow$ (ii) $\Rightarrow$ (iii) and (i) $\Rightarrow$ (iv) are straight forward.

References

- [1].U. Acar, F. Koyuncu, B. Tanay, Soft sets and soft rings, Computers and Mathematics with Applications, 59 (11) (2010) 3458-3463.
- [2].J. Ahsan, R. Latif, M. Shabir, Representations of weakly regular semirings by sections in a presheaf, Communications in Algebra, 21 (8) (1993) 2819-2835.
- [3].H. Aktas, N. Cagman, Soft sets and soft groups, Information sciences, 177 (13) (2007) 2726-2735.
- [4].M. I. Ali, M. Shabir, K. P. Shum, On soft ideals over semigroups, Southeast Asian Bulletin of Mathematics, 34 (2010) 595-610.
- [5].M. I. Ali, F. Feng, X. Liu, W. K. Min, M. Shabir, On some new operations in soft set theory, Computers and Mathematics with Applications, 57 (2009) 1547-1553.
- [6].M.I. Ali, M. Shabir, M. Naz, Algebraic structures of soft sets associated with new operations, Computational and Applied Mathematics with Applications, 61 (2011) 2647-2654.

- [7]. S. K. Bhambri, M. K. Dubey, Anuradha, On Prime, Weakly Prime Left Ideals and Weakly Regular Ternary Semirings, *South Asian Bulletin of Mathematics*, 37 (2013) 801-811.
- [8]. N. Cagman and S. Enginoglu, Soft matrix theory and its decision making, *Computers and Mathematics with Applications*, 59 (2010) 3308-3314.
- [9]. N. Cagman and S. Enginoglu, Soft set theory and uni-int decision making, *European Journal of Operational Research*, 207 (2010) 848-855.
- [10]. T. K. Dutta, S. Kar, On Regular Ternary Semirings, *Advances in Algebra, Proceedings of the ICM Satellite Conference in Algebra and Related Topics*, World Scientific, (2003) 343-355.
- [11]. T. K. Dutta, S. Kar, A note on regular ternary semirings, *Kyungpook Mathematics*, 46 (2006) 357-365.
- [12]. F. Feng, Y. B. Jun, X. Zhao, Soft semirings, *Computers and Mathematics with Applications*, 56 (10) (2008) 2621-2628.
- [13]. F. Feng, Y. B. Jun, X. Y. Liu, L. F. Li, An adjustable approach to fuzzy soft based decision making, *Journal of Computational and Applied Mathematics*, 234 (2010) 10-20.
- [14]. F. Feng, M. I. Ali, M. Shabir, Soft relations applied to semigroups, *Filomat*, 27 (7) (2013) 1183-1196.
- [15]. Y. B. Jun, Soft BCK/BCI-algebras, *Computers and Mathematics with Applications*, 56 (2008) 1408-1413.
- [16]. Y. B. Jun, C. H. Park, Applications of soft sets in ideal theory of BCK / BCI-algebras, *Information Sciences*, 178 (2008) 2466-2475.
- [17]. D. H. Lehmer, A ternary analogue of abelian groups, *American Journal of Mathematics*, 59 (1932) 329-338.
- [18]. W. G. Lister, Ternary rings, *Transaction of American Mathematical Society*, 154 (1971), 37-55.
- [19]. X. Ma and J. Zhan, SOFT INTERSECTION h-IDEALS OF HEMIRINGS AND ITS APPLICATIONS, *International Journal of Pure and Applied Mathematics*, 32-2014 (301-308).
- [20]. X. Ma, J. Zhan, Applications of a new soft set to h-hemiregular hemirings via (M,N)-SI-h-ideals, *Journal of Intelligent and Fuzzy Systems*, 26 (2014) 2515-2525.
- [21]. T. Mahmood, U. Tariq, Generalized k-Ideals in Semirings using Soft Intersectional Sets, *International Journal of Algebra and Statistics*, 4 (1) (2015), 20-38.
- [22]. P. K. Maji, A. R. Roy, R. Biswas, An application of soft sets in a decision making problem, *Computers and Mathematics with Applications*, 44 (2002) 1077-1083.
- [23]. P. K. Maji, R. Biswas, A. R. Roy, Soft set theory, *Computers and Mathematics with Applications*, 45 (2003) 555-562.
- [24]. D. Molodtsov, Soft set theory ----- first results, *Computers and Mathematics with Applications*, 37 (1999) 19-31.
- [25]. Peng, Xindong, Y. Yang, Interval-valued Hesitant Fuzzy Soft Sets and their Application in Decision Making, *Fundamenta Informaticae* 141.1 (2015) 71-93.
- [26]. A. Sezgin, A. O. Atagun, On operations of soft sets, *Computers and Mathematics with Applications*, 61 (2011) 1457-1467.
- [27]. S. Z. Song, H. S. Kim, Y. B. Jun, Ideal theory in semigroups based on Intersectional Soft Sets, *The Scientific World journal*, 2014 (2014).
- [28]. H. S. Vandiver, Note on a simple type of Algebra in which cancellation law of addition does not hold, *Bulletin of the American Mathematical Society*, 40 (1934) 914-920.
- [29]. Y. Zou, Z. Xiao, Data analysis approaches of soft sets under incomplete information, *Knowledge-Based Systems*, 21 (2008) 941-945.